ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2017 SESSION

Open-book examination
Calculators: only authorized models
Duration: 3 hours

14-IF-A4 Program Design and Data Structures

Programming languages are C, C++ and Java.

1. [10 points]
   For each of the following code segments, find a function f(n) with the property that the number of statements executed by the segment is O(f(n)). Your function f(n) should be as simple as possible.
   — Segment 1:
   ```
   for(int i = 0, Sum = 0; i < n; i++)
   Sum++;
   ```
   — Segment 2:
   ```
   for (int i = 0, Sum = 0; i < n; i++)
   for (int j = 0; j < n; j++)
   Sum++;
   ```
   — Segment 3 :
   ```
   for (int i = 0, Sum = 0; i < n; i++)
   Sum++;
   for (int j = 0; j < n * n; j++)
   Sum++;
   ```
   — Segment 4 :
   ```
   for (int i = 0, Sum = 0; i < n; i++)
   for (int j = 0; j < i; j++)
   Sum++;
   ```
   — Segment 5 :
   ```
   for (int i = 0, Sum = 0; i < n; i++)
   for (int j = 0; j < i * i; j++)
   for (int k = 0; k < j ; k+)
   Sum++;
   ```

2. [10 points]
   Indicate true or false and justify.

   (a) Is $2^{n+1} = O(2^n)$?

   (b) Is $2^{2n} = O(2^n)$?

3. [10 points] Give all the preconditions, postconditions and the loop invariants for the function (method) `factorial`.

```
//-----------------------------------
// Computes the factorial of an integer.
// Precondition:
// Postcondition:
// Loop invariant:
//-----------------------------------
public static int factorial (int n) {
  int result  = 1;

  for (int i = n; i > 1; --i) {
   result *= i;
  } // end for
  return result
} // end factorial
```

4. [10 points]
   Suppose that you have a shuffled deck of 52 cards, 13 cards in each of 4 suits, and you wish to sort the deck so that the 4 suits are in order and the 13 cards within each suit are also in order. Which of the following methods is fastest? and why?

   (a) Go through the deck and remove all clubs; then sort them separately. Process to do the same for diamonds, the hearts, and the spades.

   (b) Deal the cards into 13 piles according to the rank of the card. Stack these 13 piles back together and deal into 4 piles according to suit. Stack these back together.

   (c) Make only one pass through cards, by placing each card in its proper position relative to the previously sorted cards.

5. [10 points]
   A vertex cover of a graph $G = (V, E)$ is a subset of vertices $V' \in V$ such that every edge in E contains at least one vertex from V'. Delete all the leaves from any depth-first search tree of G. Must the remaining vertices form a vertex cover of G? Give a proof or a counterexample.

6. [10 points]
   There are eight small islands in a lake, and the state wants to build seven bridges to connect them so that each island can be reached from any other

one via one or more bridges. No connection with the lake border. The cost of constructing a bridge is proportional to its length. The distances between pairs of islands are given in the following table.

	1	2	3	4	5	6	7	8
1	-	248	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-

Find which bridges to build so that the total construction cost is minimum.

Give the algorithm name that you used, and indicate how to find the solution.

7. [10 points]

   Give a code to implement a chained list, then give the code for the methods or (functions) for `eliminateDuplicates` of the list and `invert` the list.

   For the first method, the list (a, b, a, a, b, c, a) returns (a, b, c).

   For the second method, the list (a, b, c, d, e) returns (e, d, c, b, a).

8. [10 points]

   Write a recursive method (function) to calculate the sum of the n first integers of an array containing at least n integers.

   ```
   int sum( int arr[], int n ) { ... }
   ```

9. [10 points]

   Give an algorithm for finding the k smallest elements of an unsorted set of n integers in $O(n + k \log n)$.

10. [10 points]

    Use the partitioning idea of quicksort to give an algorithm that finds the median element of an array of n integers in expected $O(n)$ time.

End of exam.