ORDRE DES INGÉNIEURS DU QUÉBEC

NOVEMBER 2017 SESSION

Open-book examination
Calculators: only authorized models
Duration: 3 hours

14-IF-A4 Program Design and Data Structures

Programming languages are C, C++ and Java.

1. [10 points]
   How does "programming by contract" help to improve software quality? Explain and justify your answer.
2. [10 points]
   What is the difference between functional testing and unit testing? Explain and justify your answer.
3. [10 points]
   There are 25 horses. At most, 5 horses can race together at a time. You must determine the fastest, the second fastest, and the third fastest horse. Find the minimum number of races in which that can be done. Give detailed explanation of how you find this minimum.
4. [10 points]
   If I prove that an algorithm takes $O(n^2)$ worst-case time, is it possible that it takes $O(n)$ on all inputs? Justify your answer.
5. [10 points]
   The Grinch is given the job of partitioning $2n$ players into two teams of n players each. Each player has a numerical rating that measures how good he/she is at the game. He seeks to divide the players as unfairly as possible, so as to create the biggest possible talent imbalance between team A and team B. Explain how the Grinch can do the job in $O(n \log n)$ time.
6. [10 points]
   Given a set S containing n real numbers, and a real number x. Give an algorithm to determine whether two elements of S exist whose sum is exactly x.

(a) Assume that S is unsorted. Give an $O(n \log n)$ algorithm for the problem.

(b) Assume that S is sorted. Give an $O(n)$ algorithm for the problem.

Justify your answer.

7. [10 points]
For each of the following pairs of functions, either $f(n)$ is in $O(g(n))$, $f(n)$ is in $\Omega(g(n))$, or $f(n) = \Theta(g(n))$. Determine which relationship is correct and briefly explain why.

(a) $f(n) = \log^2 n$; $g(n) = \log n$

(b) $f(n) = n$; $g(n) = \log^2 n$

(c) $f(n) = 2^n$; $g(n) = 10n^2$

(d) $f(n) = 2^n$; $g(n) = 3^n$

8. [10 points]
Let G be a weighted directed graph with n vertices and m edges, where all edges have positive weight. A directed cycle is a directed path that starts and ends at the same vertex and contains at least one edge. Give an $O(n^3)$ algorithm to find a directed cycle in G of minimum total weight.

9. [10 points]
Using recursion, write a code to calculate the area of a polygone. The polygon can be divided in triangles, the area of a triangle defined by the points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is :

$$A = \frac{|x_1y_2 + x_2y_3 + x_3y_1 - y_1x_2 - y_2x_3 - y_3x_1|}{2}$$

10. [10 points]
Verify that this segment of a program:

```
x = 2;
z = x + y;
if ( y > 0 )
   z = z + 1;
else
   z = 0;
```

is correct for the precondition $y = 3$ and the postcondition $z = 6$.

End of exam.