ORDRE DES INGÉNIEURS DU QUÉBEC

SESSION OF NOVEMBER 2018

All documentation allowed
Calculator : only authorized models
Length of the test : 3 hours

16-MC-A3 System Analysis and Control

Question 1 (20 pts)

A schematic diagram of a quarter car with an active damping system is shown in Figure 1. The actuator exerts a force F between the wheel and the body based on feedback from the distance between the body and the center of the wheel. Let x_b, x_w and x_r represent the heights of body, wheel and road measured from their equilibria. A simple model of the system is given by

$$m_b \ddot{x}_b = F \tag{1}$$

$$m_w \ddot{x}_w = -F + k_t(x_r - x_w) \tag{2}$$

$$\ddot{x}_b = \frac{d\dot{x}_b}{dt}, \tag{3}$$

$$\dot{x}_b = \frac{dx_b}{dt}, \tag{4}$$

$$\ddot{x}_w = \frac{d\dot{x}_w}{dt}, \tag{5}$$

$$\dot{x}_w = \frac{dx_w}{dt}, \tag{6}$$

where m_b is a quarter of the body mass, m_w is the effective mass of the wheel including brakes and part of the suspension system and k_t is the tire stiffness. The active damper can be tuned, with k and c two parameters to adapt, to simulate a conventional damper :

$$F = k(x_w - x_b) + c(\dot{x}_w - \dot{x}_b) \tag{7}$$

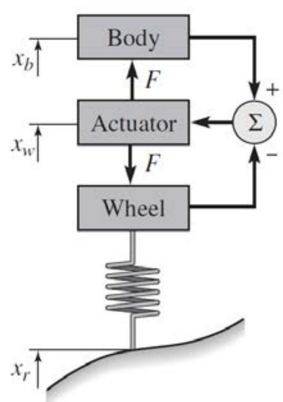

Figure 1.

Questions :

1 – Determinate the 7 Laplace Tranform of the numbered relations from (1) to (7) with null initial conditions (5 points).

2 – Draw the bloc diagram of the system of input x_r with the variables F, $\dot{x}_b$, $\dot{x}_w$, x_b and x_w. (15 points).

QUESTION 2 (20 pts)

Consider the lateral dynamics of an aircraft as described in the following block diagram (Figure 2).

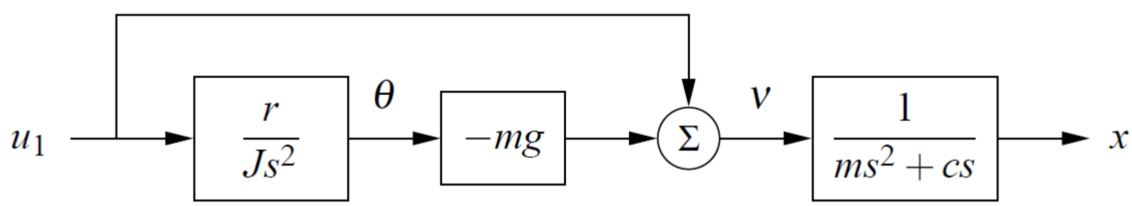

Figure 2.

Questions :

1 – Use the block diagram to compute the transfer function from $u_1(s)$ to $\Theta(s)$ (10 points)

2 – Use the block diagram to compute the transfer function from $u_1(s)$ to $x(s)$ (10 points)

QUESTION 3 (20 pts)

We consider the problem of maintaining the speed of a car with a cruise controller. The transfer function of the linearized vehicle dynamics is

$$P(s) = \frac{b}{s+a}$$

with a=0.0101 and b=1.3203.

Since the open loop dynamics is too slow, it is natural to specify a faster closed loop system by requiring that the closed loop system be of second-order with damping ratio ζ=1 and undamped natural frequency ω_0=0.2 rad/s.

Figure 3 presents the block diagrams of the closed loop system with an ideal PID controller. In this problem, only a PI controller is considered, so k_d=0.

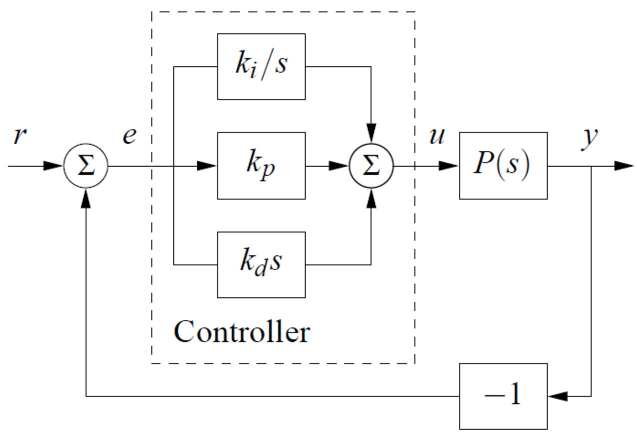

Figure 3.

<u>Questions :</u>

1 – Determinate the required characteristic polynomial of the closed loop system (5 points)

2 – Determinate the characteristic polynomial of the closed loop system with the PI controller (5 points)

3- Determinate the values of the controller parameters k_p and k_i. (10 points).

QUESTION 4 (20 pts)

Consider the dynamics for a balance system, shown in Figure 4.

If we assume that θ and $\dot{\theta}$ are small, the linearized approximation between the position p and the force F is described by the following transfer equation

$$H_{pF} = \frac{J_t s^2 - mgl}{s^2\left((M_t J_t - m^2 l^2)s^2 - M_t mgl\right)}$$

We use the parameters for the system corresponding roughly to a human being balanced on a stabilizing cart. with m=80 Kg, M_t=10 Kg, J_t=100 Kg m^2/s^2, l= 1 m, g=9.8 m/s^2.

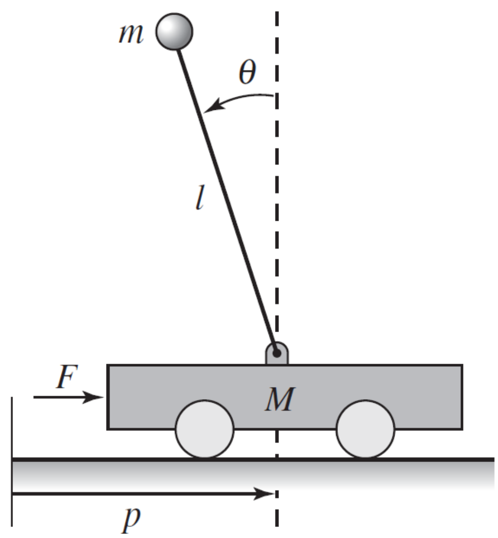

Figure 4.

<u>Questions :</u>

1 – Compute the poles and zeros of the open loop system. (5 points).

2 – Plot the pole zero diagram of Hpf. (5 points).

3- Consider the root locus method to predict the stability of the system in closed loop with a proportional gain (10 points)

QUESTION 5 (20 points)

Figure 5 presents the Bode plot and the Nyquist plot of a vectored thrust aircraft.

The system is stable in open loop.

The system has a phase margin of 68 degrees.

The system has a gain margin of 6.2.

The compensated system, $KG(s)$ with K a gain, is set in a negative closed loop.

Questions:

1- Determinate if the closed loop system with $K=+1$ is stable ? (5 points)

2- Determinate if the closed loop system with $K=+7$ is stable ? (5 points)

3- Determinate if the closed loop system with $K=+6$ is stable ? (5 points)

4- Determinate if the closed loop system with $K=-0.001$ (negative gain K) is stable ? (5 points)

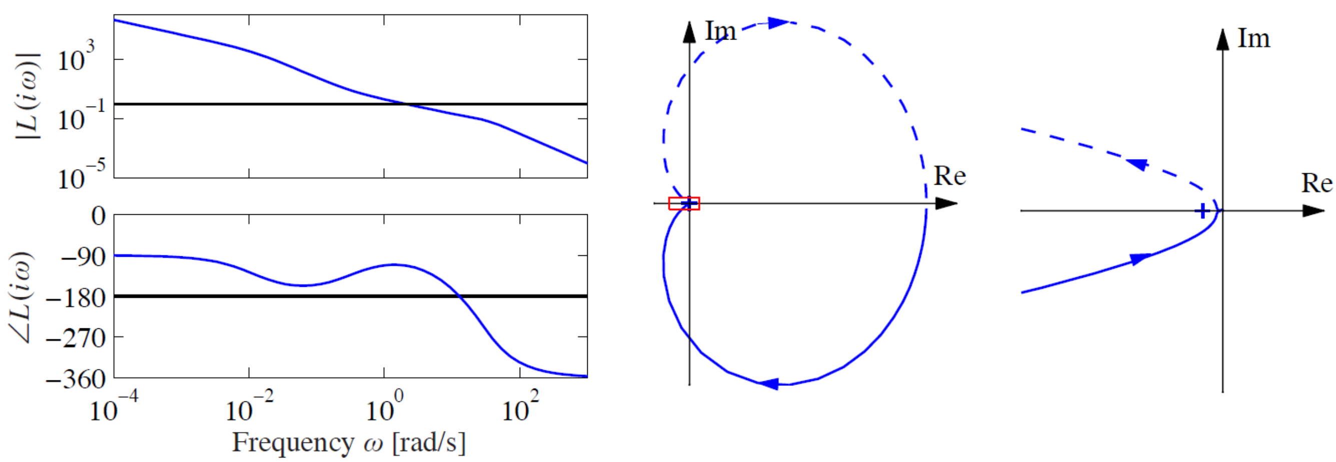

Figure 5.