ORDRE DES INGÉNIEURS DU QUÉBEC

SESSION OF MAY 2017

All documentation allowed
Calculator : only authorized models
Length of the test : 3 hours

16-MC-A3 System Analysis and Control

Question 1 (25 pts)

Figure 1 presents the bloc diagram of a flexible robot joint. The equation of the dynamics are

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{MgL}{I}\sin(x_1) - \frac{k}{I}(x_1 - x_3) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{k}{J}(x_1 - x_3) + \frac{1}{J}u
\end{aligned}$$

with the variables

$$x_1 = q_1 \qquad x_2 = \dot{q}_1$$
$$x_3 = q_2 \qquad x_4 = \dot{q}_2$$

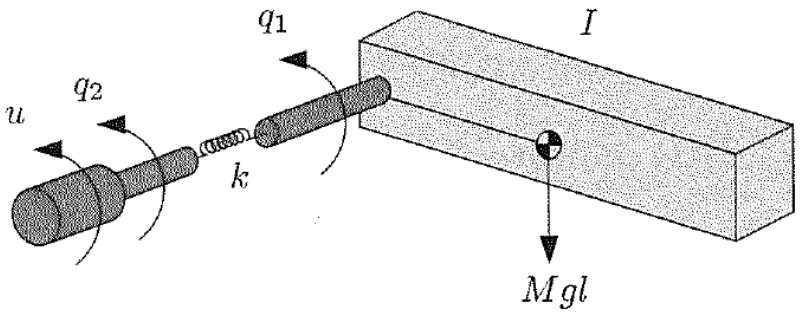

Figure 1.

Questions :

1 – Determinate the expressions of the linearized equations by considering that the angle q_1 is closed to 0 rad. (5 points).

2 – Determinate the expressions of the Laplace transforms of the 4 equations of the dynamics for null initial conditions. (5 points).

3 – Draw the causal bloc diagram with clearly noted on it : all signals ($x_1(s)$, $x_2(s)$, $x_3(s)$, $x_4(s)$ et $u(s)$), all parameters (I, l, J, M , g et k) and the variable Laplace (s) ? (15 points).

QUESTION 2 (25 pts)

We consider the control of a servo system (Figure 3) with

$$H(s) = \frac{1}{s(s+1)(s+5)}$$

The compensator is a PID :

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right).$$

The reference is a unit step signal :

$$r(t) = \begin{cases} 0 \ \ for\ t < 0 \\ 1 \ \ for\ t \geq 0 \end{cases}.$$

The final position error is defined as :

$$e_\infty = \lim_{t\to\infty} e(t)$$

.

Question :

1 – Calculate the transfer function $Y(s)/R(s)$ for $C(s)=K_p$, $T_D=0$ and $1/T_I=0$. (5 points)

2 – Calculate the critical value of K_p which ensures that the closed loop is unstable. Calculate the critical period of oscillation which corresponds to the critical gain. (15 points)

3 – By considering the Ziegler and Nichols tuning based on the critical gain and the critical period, calculate the values of the PD gain, which are K_p, T_D and T_i . (5 points).

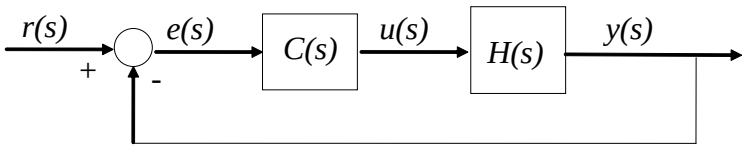

Figure 3.

QUESTION 3 (25 pts)

Figure 3 presents the block diagram of a positioning control system of reference $R(s)$ and output $Y(s)$.

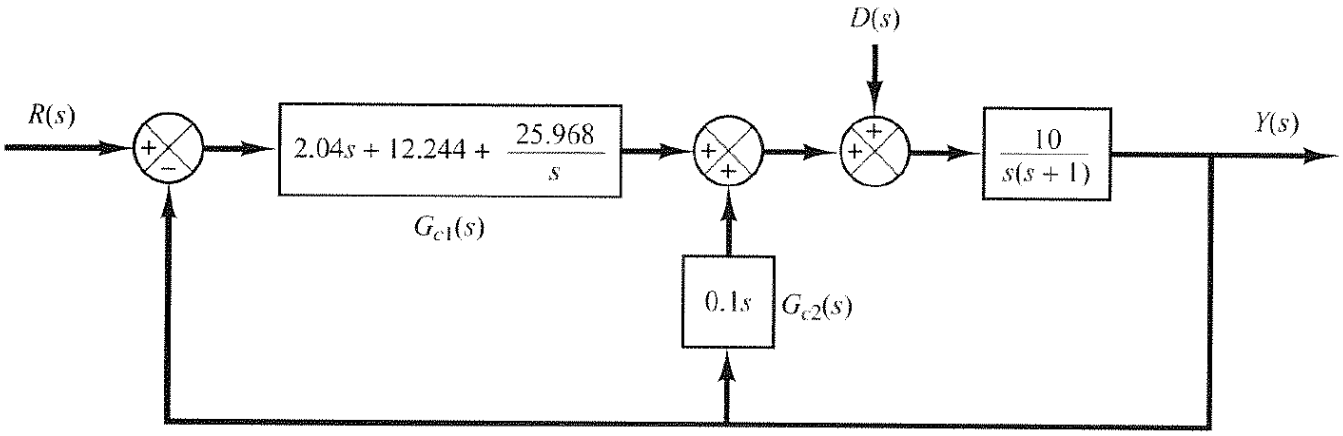

Figure 3.

Questions :

1 – Determine the transfer function Y(s)/R(s) when $D(s)=0$? (15 points).

2 – Determine that the steady-state error of position is null in the case of a unit step reference $r(t)=1$ and $D(s)=0$. (10 points).

QUESTION 4 (25 points)

Figure 4 presents the Bode plots of chemical system $G(s)$ in open-loop.

$$G(s) = \frac{Ke^{-Rs}}{s(s+a)(s+b)}$$

with R=0.1, a=4 and b=10.

Questions:

1- By considering the response at 0.2 rad/s, find the gain value of K. (15 points).

2- Is the system $G(s)$ stable in open loop ? (5 points)

3- By reading the graph, estimate the gain margin. (5 points).

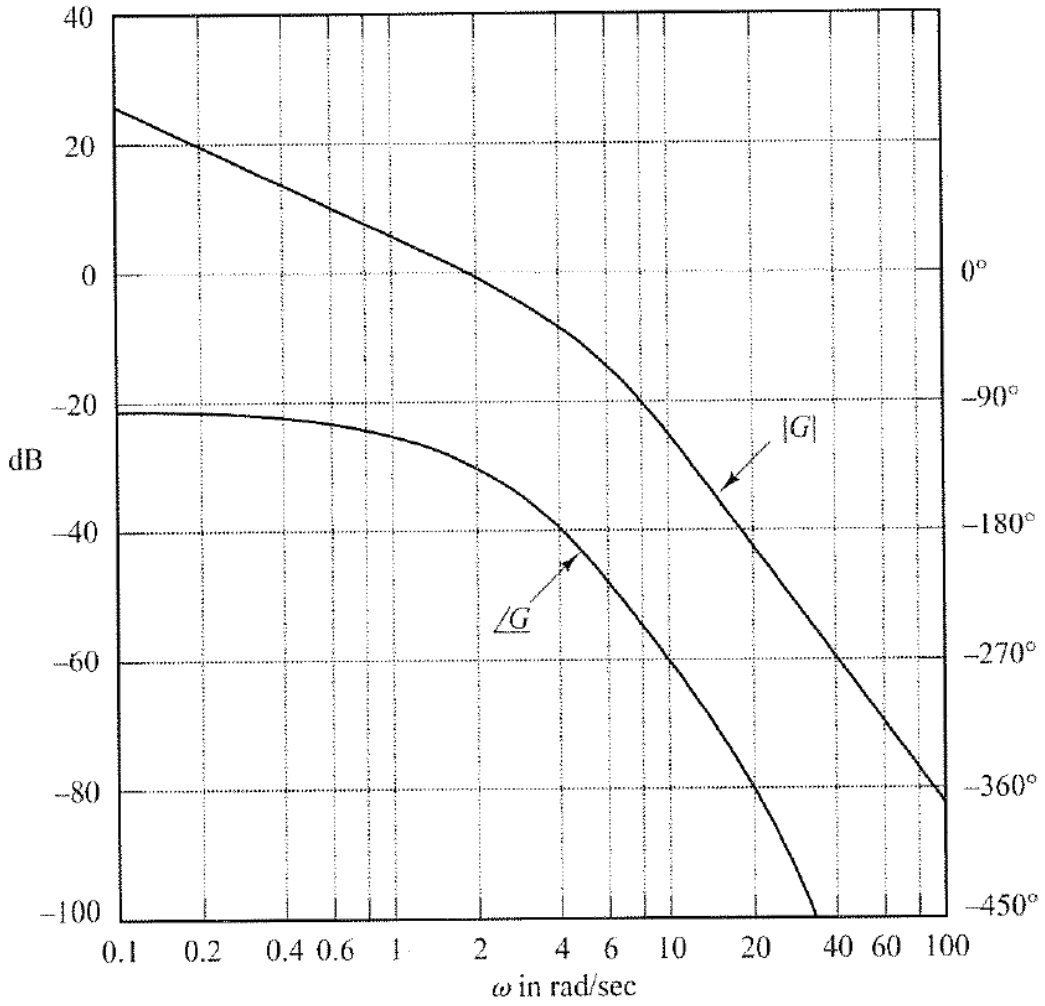

Figure 4.