ORDRE DES INGÉNIEURS DU QUÉBEC

NOVEMBER 2017 SESSION

All documentation allowed
Calculator : only authorized models
Length of the test : 3 hours

16-MC-A3 System Analysis and Control

Question 1 (20 pts)

Figure 1 shows a schematic diagram of a hydraulic damper.

We assume that the system can be described with

$$A(P_1 - P_2) = kz$$

$$q = \frac{P_1 - P_2}{R}$$

$$\frac{dy}{dt} - \frac{dz}{dt} = \frac{q}{A\rho}$$

with A the piston area, k the spring constant, R the resistance to flow at the restriction, P_1 the pressure existing on the right side of the piston, P_2 the pressure existing on the left side of the piston, y the displacement of the piston, z the displacement of the spring and q the flow at the restriction.

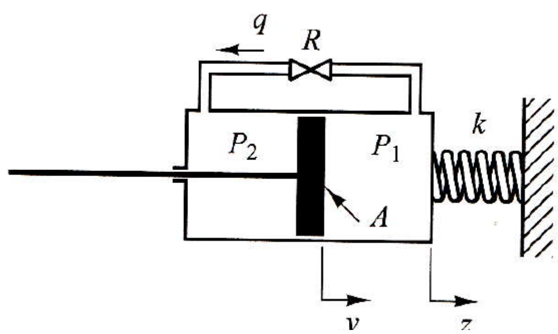

Figure 1.

<u>Questions :</u>

1 – Determinate the expressions of the Laplace transforms of the 3 equations. (5 points).

2 – Draw the causal bloc diagram with clearly noted on it : all signals (Q(S), the input $Y(s)$, $P_1(s)$ - $P_2(s)$ and the output $Z(s)$), all parameters (R,A, ρ, k) and the variable Laplace (s) ? (15 points).

QUESTION 2 (20 pts)

We consider the control of a servo system (Figure 2) with a PID compensator.

The specifications require that the response to the unit-step disturbance be such that the closed-loop poles are: 1 pole at -10, 2 complex conjugate poles with $\zeta=0.5$ (damping) and $\omega_n=4$ rad/s (natural pulsation).

<u>Question :</u>

1 – Calculate the characteristic equation of $C(s)/D(s)$ for $R(s)=0$. (5 points)

2 – Calculate the characteristic equation of the specified closed loop system. (5 points)

3 – Calculate the transfer function of the PID controller. (10 points).

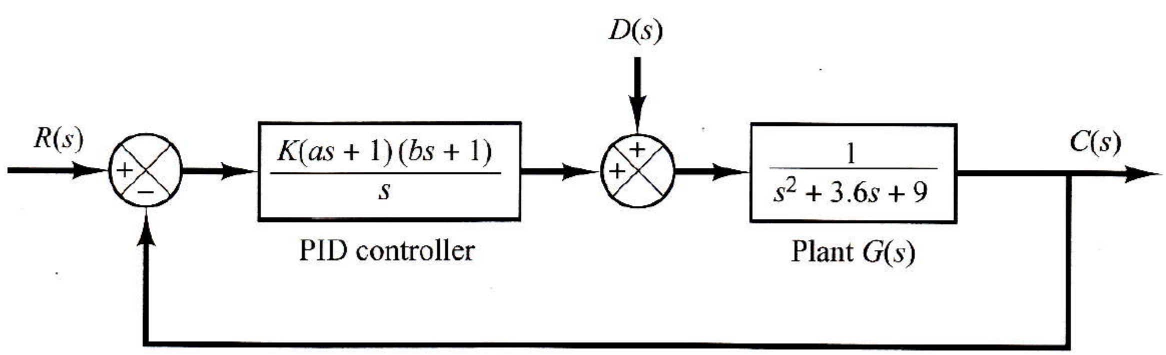

Figure 2.

QUESTION 3 (20 pts)

Figure 3 presents the block diagram of a compensator of reference $E(s)$ and output $Y(s)$.

Under normal operation of the system, we have

$$\left| \frac{a}{a+b} \frac{K}{s} \frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1} \right| \gg 1$$

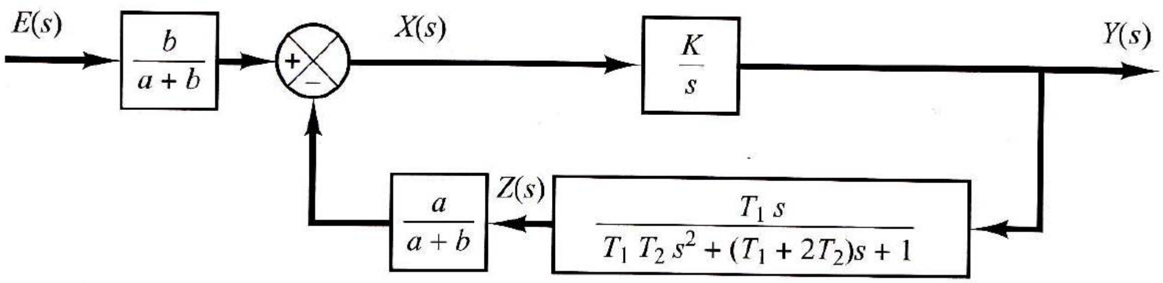

Figure 3.

<u>Questions :</u>

1 – Determine the transfer function $Y(s)/E(s)$? (5 points).

2 – Demonstrate that the system $Y(s)/E(s)$ is equivalent to a PID controller. (10 points).

3 – Give the expression of the gains K_p, K_i and K_d function of T_1, T_2 , b and a. (5 points).

QUESTION 4 (20 pts)

Figure 4 presents the root-locus branch corresponding to a system composed of 3 sub-systems in series noted k=0, 1, and 2.

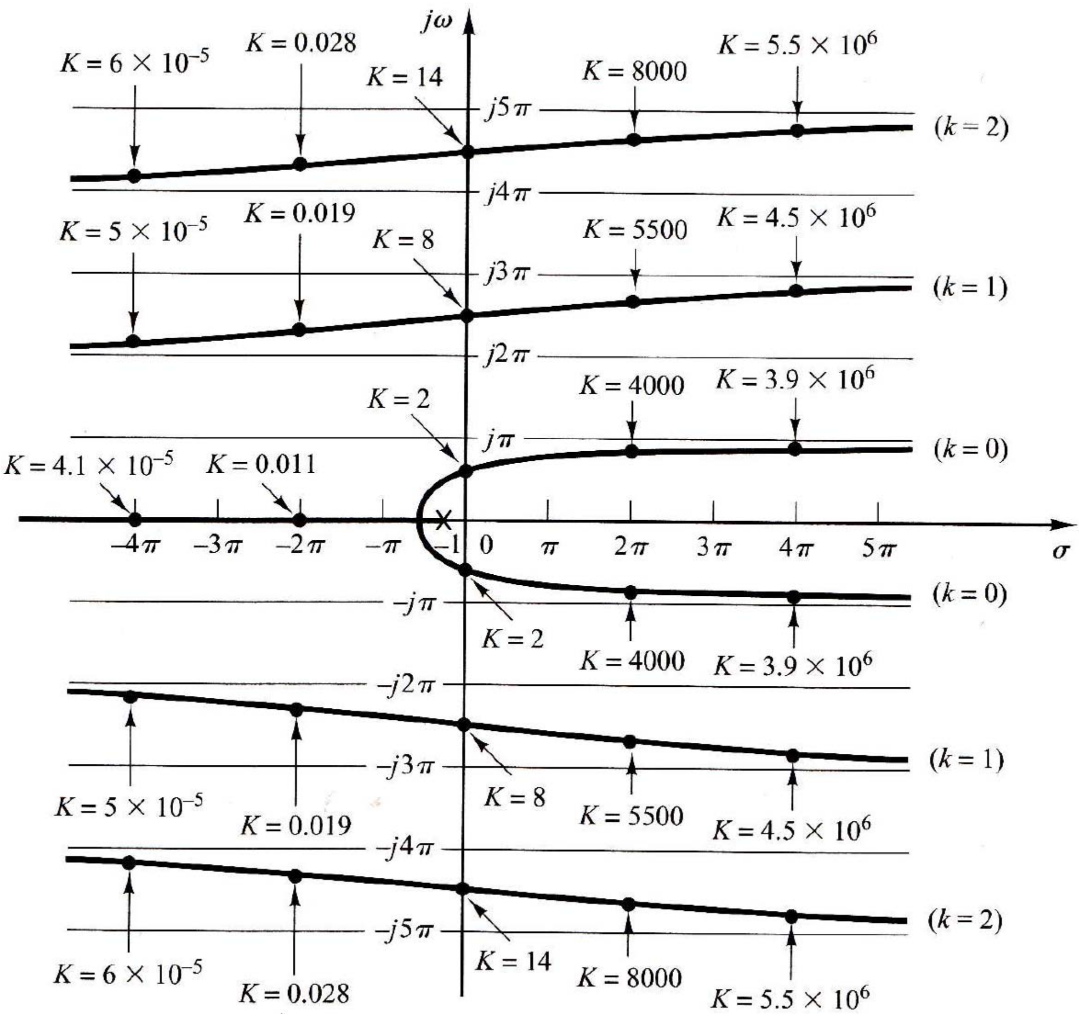

Figure 4.

<u>Questions :</u>

1 – Determine the stability of the system in closed loop for K=0.028 (5 points).

2 – Determine the stability of the system in closed loop for K=8 (5 points).

3 – Determine the stability of the system in closed loop for K=2 (5 points).

3 – Determine the pole of the sub-system 0 (k=0) in open loop. (5 points).

QUESTION 5 (20 points)

Figure 5 presents the Bode plots of a system $K(s)G(s)$ in open-loop with $K(s)=1$.

<u>Questions:</u>

1- By reading the graph, give the phase margin and the gain margin of $G(s)$. (5 points).

2- For $K(s)=2$, what is the gain margin of $K(s)G(s)$? (5 points)

3- For $K(s)=0.4426/s$, estimate the phase margin of $K(s)G(s)$? (10 points)

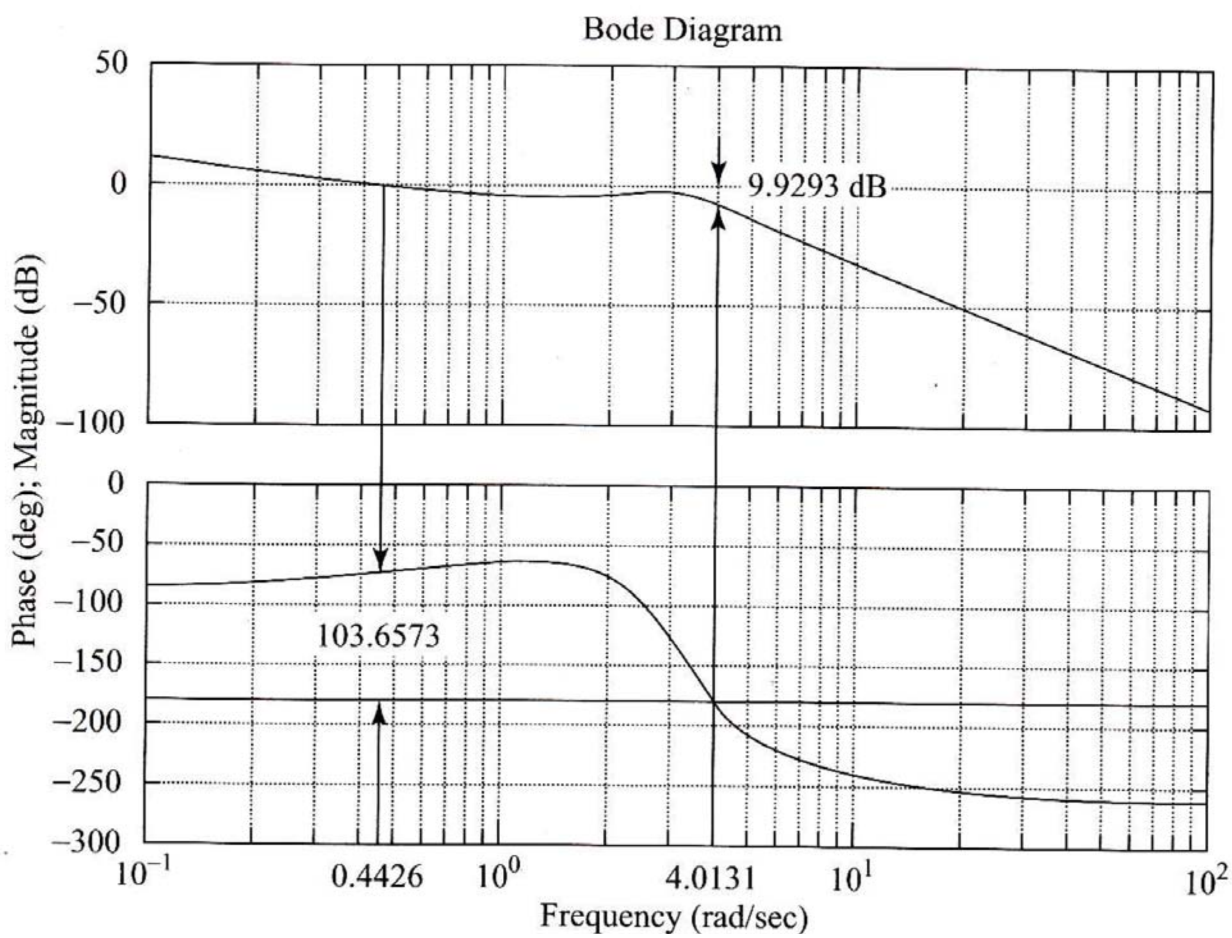

Figure 5.