ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2020 SESSION

Open-book examination
Programmable calculators, communicating calculators and personal computers are not allowed
Duration : 3 hours

16-EL-A3 Signals and Communications

Question 1 : **(20 points)**

The binary sequence $\mathbf{m} = [m_k]_{k=1,\ldots} = [1,\ 0,\ 1,\ 0,\ 0,\ 1,\ 1,\ 1,\ 0,\ 0,\ 1,\ 0]$ is applied to the input of a DPSK (Differential Phase Shift Keying) modulator with $c_k = c_{k-1} \oplus m_k$, where $c_0 = 1$, $\theta_0 = \pi$ and :

$$s_{DPSK}(t) = \cos\left(2\pi f_c t + \theta_k\right) \text{ where } \theta_k = \begin{cases} 0, & \text{for } c_k = 0 \\ \pi, & \text{for } c_k = 1 \end{cases}$$

a) Give the corresponding differentially encoded binary sequence $\mathbf{c} = [c_k]_{k=1,\ldots,12}$.

b) Give the corresponding phase sequence $[\theta_k]_{k=1,\ldots,12}$.

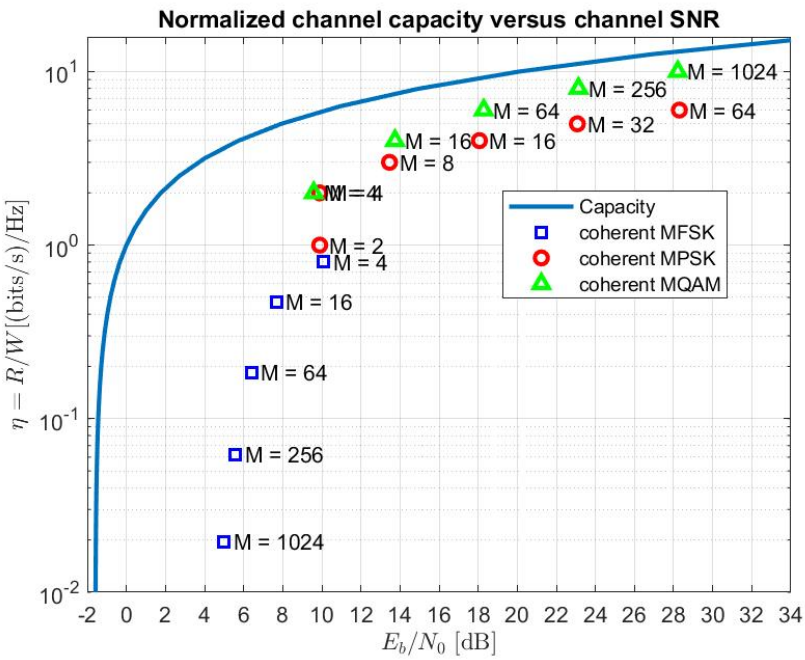
c) Draw the resulting DPSK signal waveform. For this assume that the carrier frequency is exactly $f_c = \frac{1}{T_b}$ where T_b is the bit duration.

d) Give an advantage of DPSK modulation compared to BPSK modulation.

e) Give a disadvantage of DPSK modulation compared to BPSK modulation.

Question 2 : **(20 points)**

The figure below shows the spectral efficiency plane for a symbol error probability $P_e = 10^{-5}$. It represents the spectral efficiency $\eta = R/W$ where R is the bit rate in [bits/second] and W is the bandwidth in Hertz, as a function of the signal-to-noise ratio SNR expressed as the ratio of the energy per bit E_b in [Joules] and the power spectral density of the noise N_0 in [Watt/Hertz].



a) The curve shows the capacity as a function of the signal-to-noise ratio. Explain in your own words, what is the capacity.

b) For MPSK modulation, spectral efficiency increases with higher dimensional signals M but for MFSK it decreases. Explain clearly why this is the case.

c) A communication system must be designed for which the power is limited. What would be the preferred method of modulation ? Justify your answer.

d) Is it theoretically possible to design a communication system with spectral efficiency of 8 bits/s/Hz allowing a low probability of error per symbol with a signal-to-noise ratio $E_b/N_0 = 4$ dB ? Justify your answer.

Question 3 : **(20 points)**

We want to compare the digital audio formats used for digital audio discs (CD) and very high fidelity DVD-audio discs (DVD-A). Two signals (left and right channels for stereo) are sampled each at 44.1 kilosamples per second for CDs whereas for DVD-As they are sampled at 192 kilosamples per second. An analog-to-digital converter quantizes the samples and encode them as 16-bit words for CDs and as 24-bit words for DVD-As.

a) Knowing that the signal to quantizing noise ratio for a sinewave over the whole dynamic range is $SQNR = \frac{3}{2}L^2$ (linear), L being the number of quantizing levels, what is the numerical value of the signal to quantizing noise ratio in dB, i.e. $SQNR_{dB}$ for a CD and for a DVD-A ?

b) What is the resulting bit rate R_b for both cases (CD and DVD-A) ?

c) The system must be able to record one hour of music in stereo. What is the total number of Bytes written on the CD and on the DVD-A ?

Question 4 : **(20 points)**

Consider the following modulated signal $s(t)$:

$$s(t) = A_c \left[m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t \right].$$

Write the following expressions :

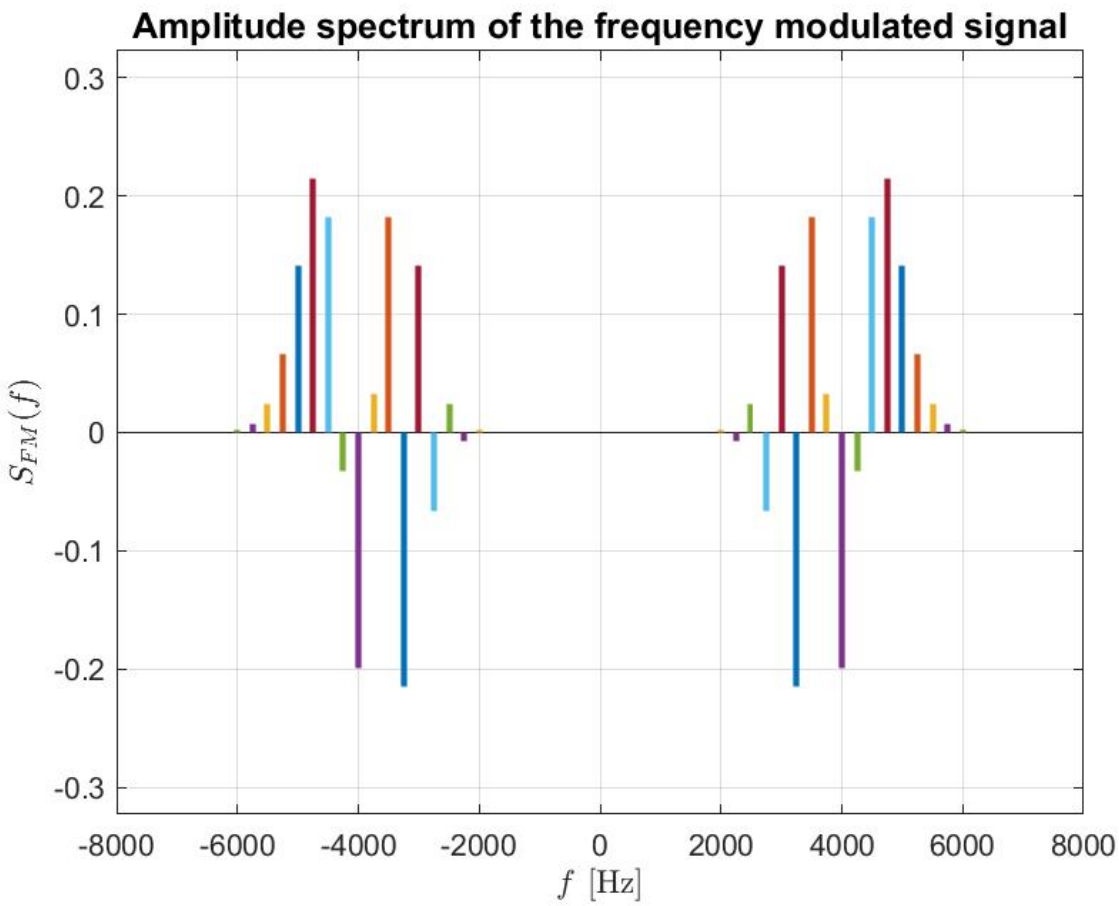
a) complex baseband representation $\tilde{s}(t)$,

b) in-phase and quadrature components $s_I(t)$ and $s_Q(t)$, as well as

c) module and phase $|s(t)|$ and $\angle s(t)$.

Question 5 : **(20 points)**

An FM signal is modulated with message $m(t) = A_m \cos(2\pi f_m t)$. The amplitude spectrum, $S_{FM}(f)$, is shown below :



a) What is the carrier frequency f_c ?

b) What is the message frequency f_m ?

c) What is the FM modulation index β_{FM} ? Is it a narrowband or a wideband signal ?

d) Using Carson's rule, estimate the transmission bandwidth, B_T, of the FM signal.

e) What is the power, P_{FM}, of this FM signal ?

Bessel function (for $0 \leq n \leq 10$)

n	$J_n(0)$	$J_n(1)$	$J_n(2)$	$J_n(3)$	$J_n(4)$	$J_n(5)$	$J_n(6)$	$J_n(7)$	$J_n(8)$	$J_n(9)$	$J_n(10)$
0	1	0.7652	0.2239	-0.2601	-0.3971	-0.1776	0.1506	0.3001	0.1717	-0.0903	-0.2459
1		0.4401	0.5767	0.3391	-0.0660	-0.3276	-0.2767	-0.0047	0.2346	0.2453	0.0435
2		0.1149	0.3528	0.4861	0.3641	0.0466	-0.2429	-0.3014	-0.1130	0.1448	0.2546
3		0.0195	0.1289	0.3091	0.4302	0.3648	0.1148	-0.1676	-0.2911	-0.1809	0.0584
4		0.0025	0.0340	0.1320	0.2811	0.3912	0.3516	0.1578	-0.1054	-0.2655	-0.2196
5			0.0070	0.0430	0.1321	0.2611	0.3621	0.3479	0.1858	-0.0550	-0.2341
6			0.0012	0.0114	0.0490	0.1310	0.2458	0.3392	0.3376	0.2043	-0.0145
7				0.0025	0.0152	0.0534	0.1296	0.2336	0.3206	0.3275	0.2167
8					0.0040	0.0184	0.0565	0.1280	0.2235	0.3051	0.3179
9						0.0055	0.0212	0.0589	0.1263	0.2149	0.2919
10						0.0015	0.0069	0.0235	0.0608	0.1247	0.2075
11							0.0020	0.0083	0.0256	0.0622	0.1231
12								0.0027	0.0096	0.0274	0.0634
13									0.0033	0.0108	0.0290
14									0.0010	0.0039	0.0120
15										0.0013	0.0045
16											0.0016