ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2019 SESSION

Open-book examination

Calculators : only authorized models

Duration : 3 hours

16-EL-A2 SYSTEMS AND CONTROL

Question 1 (5% + 5% + 5% + 5% = 20%)

Two transfer functions $G_1(s)$ and $G_2(s)$ are cascaded in the forward path of a unity feedback system (Figure 1).

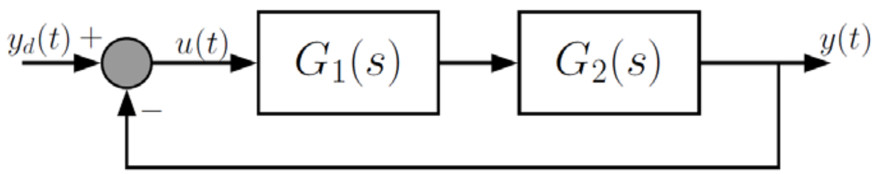

Figure 1: Block diagram for question 1

The characteristics of these two transfer functions are shown in figures 2 and 3:

1) $G_1(s)$: See the Bode diagram shown in figure 2.
2) $G_2(s)$: See the Bode diagram shown in figure 3. The plateau is at around -26.0 dB.

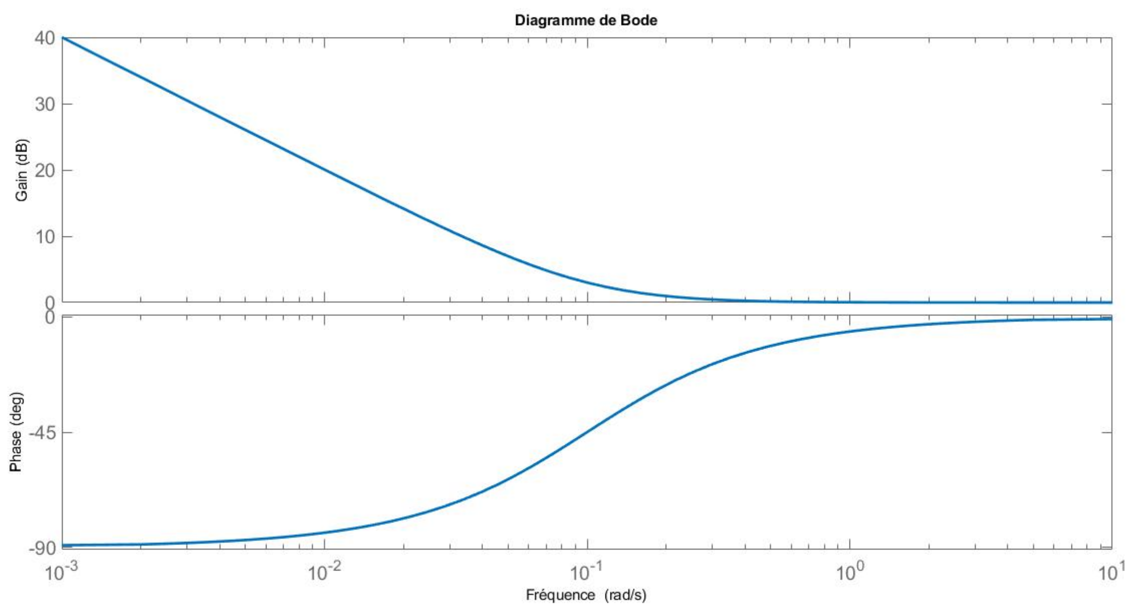

Figure 2: Bode Diagram of $G_1(s)$

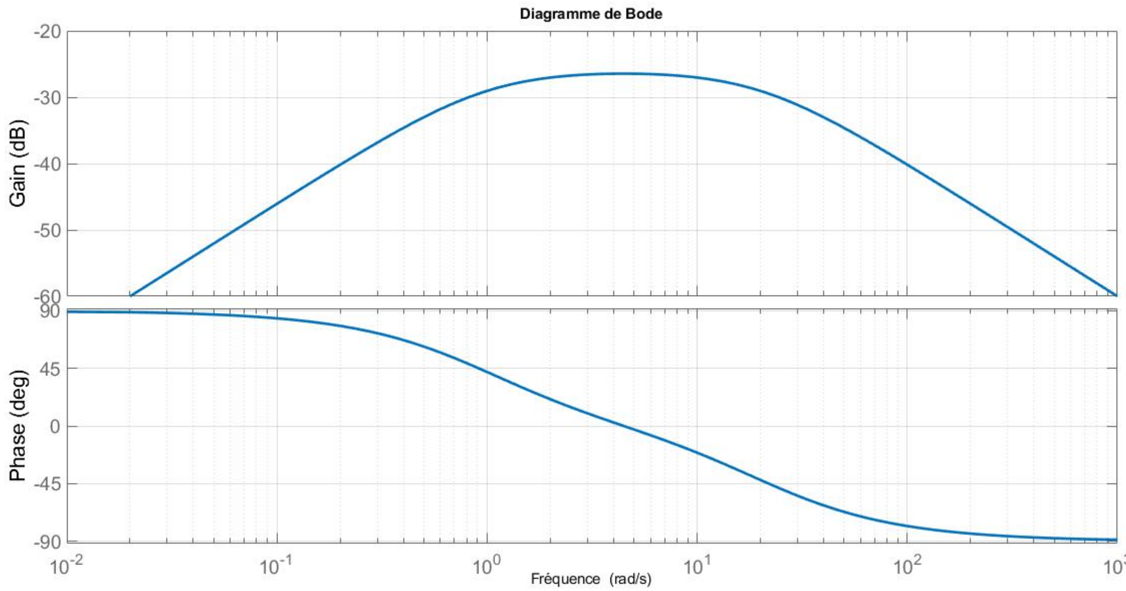

Figure 3: Bode diagram of $G_2(s)$

Answer the following questions:

a) Determine the transfer function $G_1(s)$. Explain how you find it.

b) Determine the transfer function $G_2(s)$. Explain how you find it.

c) Determine the closed-loop transfer function of the system;

d) Apply Routh's criterion (give the table) to determine whether the system is stable?

Question 2 (5% + 5% + 5% + 5% = 20%)

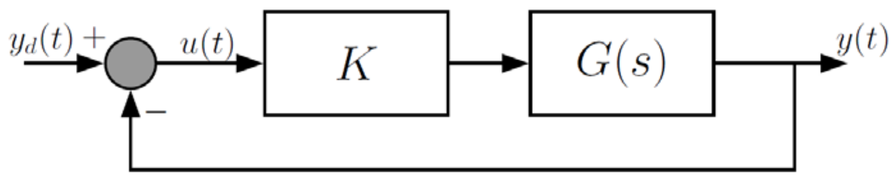

Figure 4: Block diagram of question 2

Sketch on a full page of your examination book, the root locus of the system shown in Figure 4 which has the following open loop transfer function:

$$KG(s) = \frac{K(s+1)}{(s^2+2s+2)(s+5)} \tag{1}$$

For this purpose, you **probably** need to calculate:

a) The coordinates of the point where the asymptotes meet on the real axis and the angle of each asymptote with respect to the real axis;
b) The angle of departure/arrival of complex poles and zeros;
c) The coordinates of the connection/separation point with the real axis of the branches tending to infinity;
d) Provide the range of gain K (≥0) for which the system is stable? Give the poles location of the closed loop system at the critical gain.

Question 3 (5% + 5% + 5% + 8% +2% = 25%)

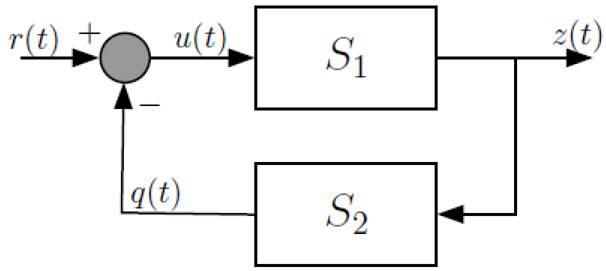

Figure 5: Block diagram for question 3

Consider the feedback controlled system as the one depicted in Figure 5 where the subsystems S_1 and S_2 can be modeled as follow:

$$S_1: \quad 5\frac{d^2z(t)}{dt^2}+1\frac{dz(t)}{dt}+5z(t)=u(t) \tag{2}$$

and :

$$S_2: \quad \frac{d^2q(t)}{dt^2}+2\frac{dq(t)}{dt}+q(t)=2z(t)\ . \tag{3}$$

a) Give a state space representation of sub-system S_1;
b) Give a state space representation of sub-system S_2;
c) Give the transfer function of the complete system: G(s) = Z(s)/R(s), considering R(s) and Z(s) as the Laplace transforms of r(t) and z(t) respectively.;
d) Give the commandable state-space model of the complete system G(s);
e) If a unit step is applied at the input r(t) of this system, what will be the steady state value that could be observed at the output z(t)?

Question 4 (3.5% + 3.5% + 3% = 10%)

Consider the Nyquist's diagram given in Figure 6 (Figure 7 shows a zoom of the diagram in the neighborhood of (-1,0)) :

a) From this diagram, find the gain margin (Sketch this diagram on a page of your exam book and show how you have found this value).
b) From this diagram, find the phase margin (Sketch this diagram on a page of your exam book and show how you have found this value).
c) Discuss about the stability of the closed-loop system.

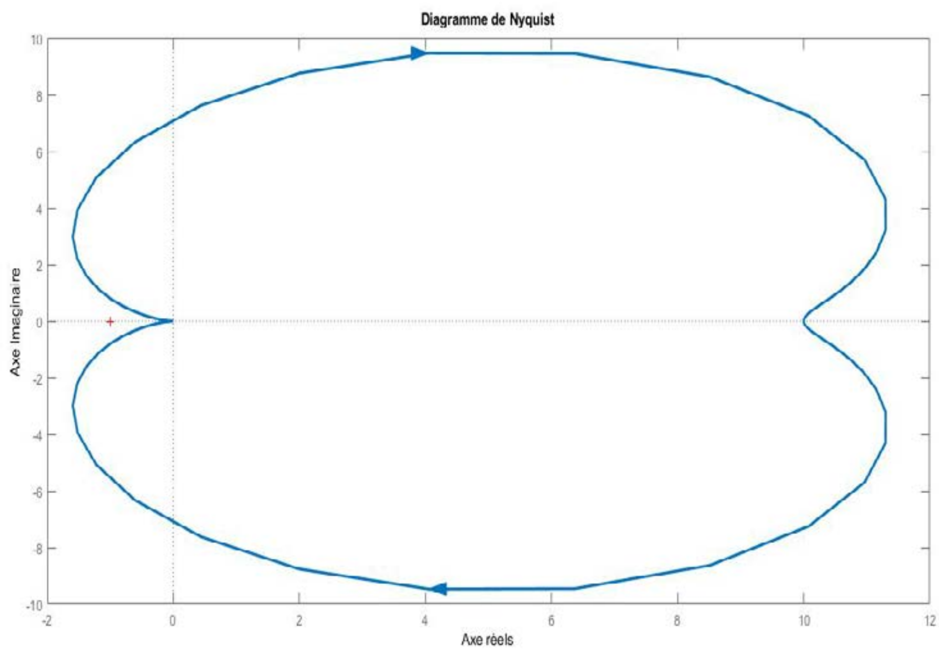

Figure 6: Nyquist's diagram for question 4

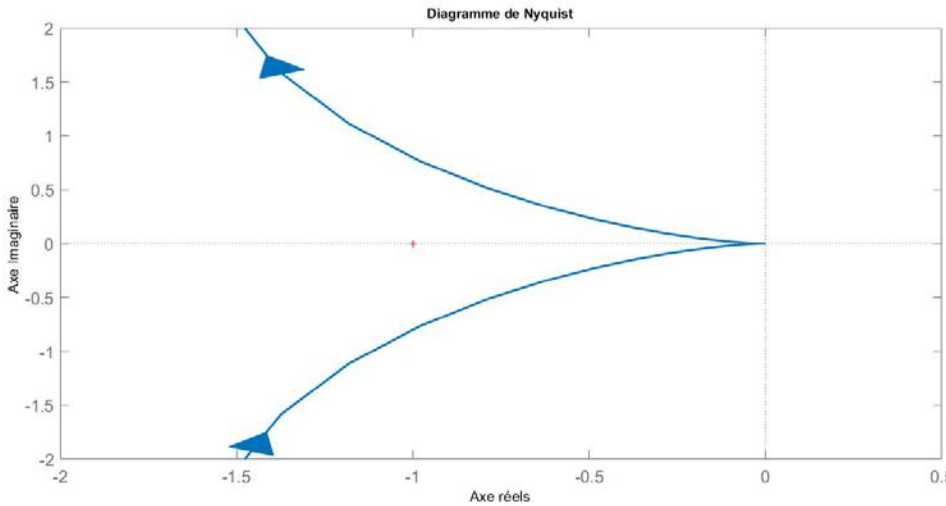

Figure 7: Zoom of the Nyquist's diagram in the neighborhood of (-1,0)

Question 5 (5% + 2% + 4% + 14% = 25%)

Consider the controlled system shown in Figure 8, where:

$$G(s) = \frac{10}{s(s+15)} \tag{4}$$

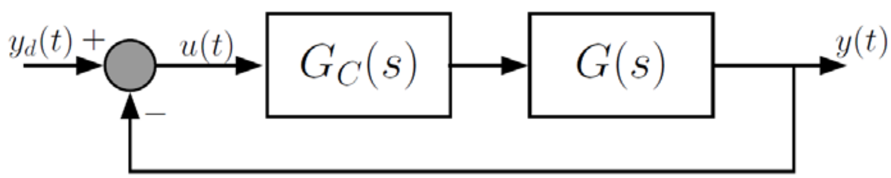

Figure 8: Block diagram for question 5

a) Compute the velocity error constant and the velocity error of the system (for an input ramp of amplitude 1). At this step, consider $G_C(s) = 1$.
b) We would like to reduce the position error by a factor of 100, and $G_C(s)$ is a pure gain, compute the gain value.
c) We would like to reduce the position error by a factor of 100 while obtaining a phase margin of 45°. Which type of controller $G_C(s)$ should be used in order to achieve such performance?
d) Compute the parameters of $G_C(s)$ which guarantee the performance described in c). Clearly detail each step of your design approach.