ORDRE DES INGÉNIEURS DU QUÉBEC

MAI 2020 SESSION

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| Open-book examination<br>Calculators : only authorized models<br>Duration : 3 hours |

**16-EL-A2 SYSTEMS AND CONTROL**

## Question 1 (10% + 5% + 5% + 5% = 25%)

Let's define a dynamic system having a pole and a zero represented by the transfer function of the following form:

$$G(s) = K\frac{(s+a)}{(s+b)}$$

and whose Nyquist diagram appears in Figure 1.

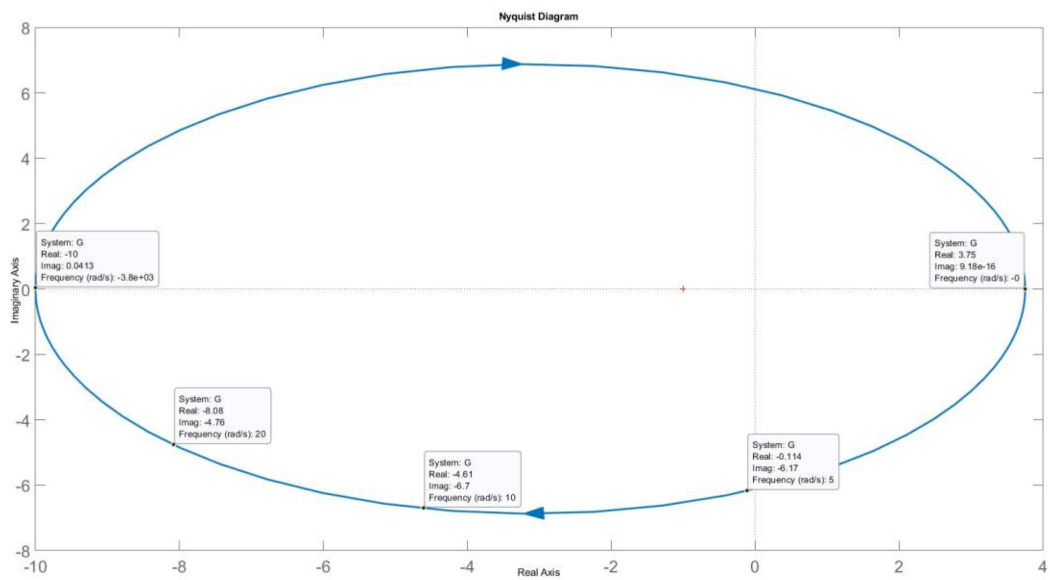


**Figure 1: Nyquist diagram of the question #1 system**

To help you, the 5 values obtained from this diagram are listed in the following table:

| Frequency rad/s | Real part of G(jω) | Imaginary part of G(jω) |
|---|---|---|
| **0 rad/s** | 3.750 | 0.000 |
| **∞ rad/s** | -10.000 | 0.000 |
| **5 rad/s** | -0.114 | -6.170 |
| **10 rad/s** | -4.610 | -6.700 |
| **20 rad/s** | -8.080 | -4.760 |

a) Obtain the transfer function of this dynamic system (i.e., calculate the values of $K$, $a$ and $b$). Explain the method followed to obtain this function.

b) Calculate the output in the time domain $y(t)$ of this system when the input is an amplitude step 10. Explain the method followed to obtain the output equation.

Let's add a proportional gain control $K_P$ to the system described above (Figure 2).

c) Obtain the closed loop system transfer function.
d) For which gain values $K_P$ the closed loop system is stable.

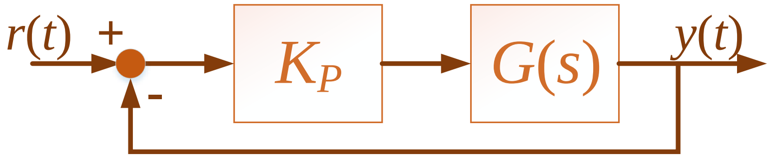


**Figure 2: Closed loop system with proportionnal control (Question #1)**

## Question 2 (7 % + 2.5% + 3 % + 2.5 % + 5% + 5% + root locus : 10% = 35%)

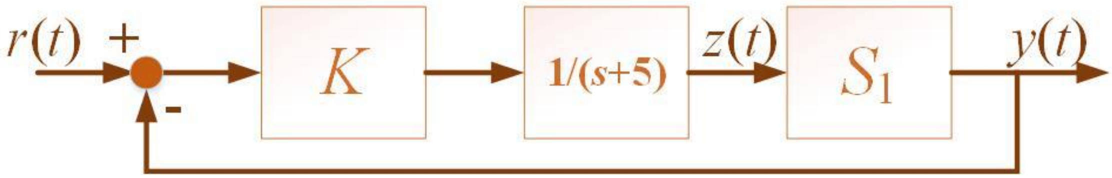


**Figure 3: Block diagram of question 2**

It is requested to draw, on a full page, the roots locus of the system (shown in Figure 3), but first you have to get **the open loop transfer function**.

a) The subsystem $S_1$ is expressed by the following state equation :

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -5 & -2 \\ 2 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} z(t)$$
$$y(t) = \begin{bmatrix} 3 & 1 \end{bmatrix} \boldsymbol{x}(t)$$

Get the transfer function $G_1(s) = Y(s)/Z(s)$ of the subsystem $S_1$, considering that Z(s) and Y(s) respectively represent the Laplace transform of z(t) and y(t).

b) Obtain the open loop transfer function of the system shown in Figure 3. This transfer function will be used later.

Trace the roots locus of the open loop transfer function obtained in b). To get it, you will **probably** need to calculate:

c) The coordinate of the point of intersection of the asymptotes, as well as the angle of these asymptotes with respect to the real axis;
d) The angle of departure and arrival of the poles and complex zeros;
e) The coordinates of the branches' disconnection / connection points with the real axis;
f) For which gain range $K$ will the system be stable in closed loop, with unitary feedback? For the critical gain value, what will be the position of the poles of the closed-loop system, still with unitary feedback?
g) **Draw the root locus.**

## Question 3 (8% + 7% + 3% + 2% = 20%)

Let's start with the following transfer function:

$$G(s) = \frac{10(-s+1)}{(s+2)(s+5)}$$

a) Calculate the module (amplitude) and the argument (phase) of the function G(jω).
b) Draw the Black's diagram on the Black-Nichols chart provided on the next page (attach this page to your exam book).
c) From the diagram, conclude on the stability of the closed loop system.
d) What is the system gain margin?

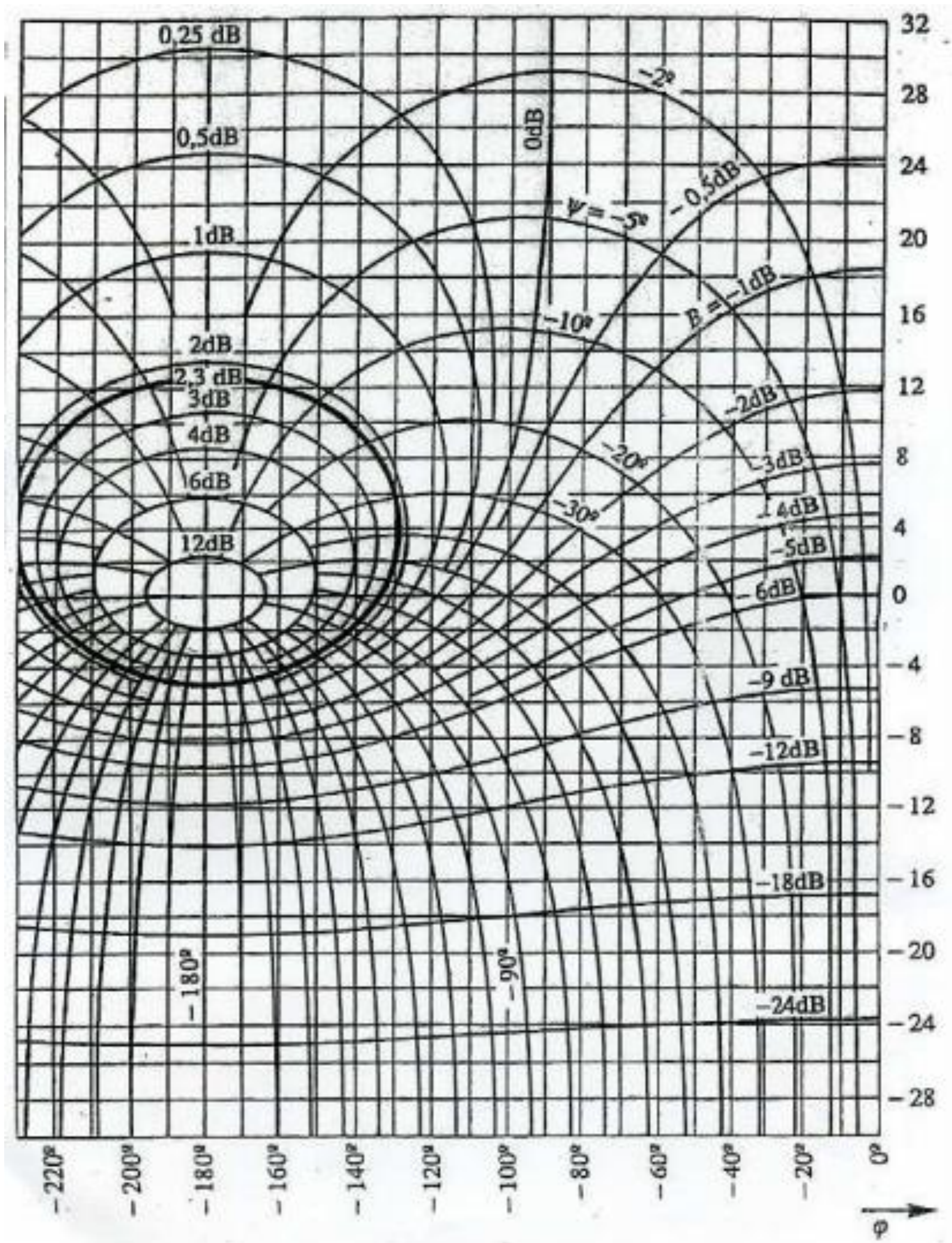


**Figure 4: Black-Nichols abacus**

## Question 4 (5% + 2.5% + 12.5% = 20%)

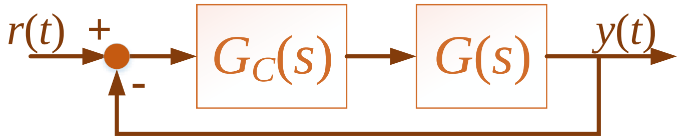


**Figure 5: Block diagram of the system of question #5**

Let's start with the control system shown in Figure 5 and having the following transfer function G(s):

$$G(s) = \frac{50}{(s+50)(s+1)} \tag{1}$$

a) Considering a proportional integral compensator defined by $G_C(s) = K_C(1 + 10/s)$, obtain the gain $K_C$ which ensures that the transient response have an overshoot of 10%. The Bode diagram of the transfer function $\frac{50(1+10/s)}{(s+50)(s+1)}$ is shown in Figure 6 to assist you in the design. Clearly detail each step of the design process.

b) What is the speed error constant corresponding to the compensator designed in a)?

c) Considering now that we add in series with $G_C(s)$ a lag delay compensator, design this compensator to ensure a 10% overshoot and a speed error constant 10 times higher than that obtained in b) ? Clearly detail each step of the design process.

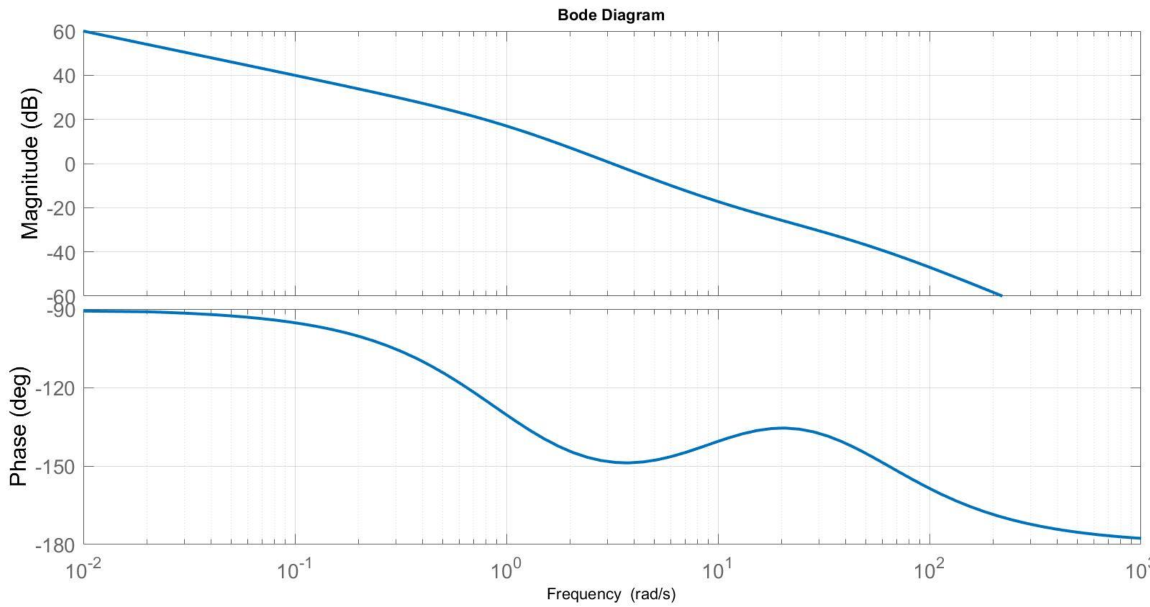


**Figure 6: Bode diagram of $\frac{50(1+10/s)}{(s+50)(s+1)}$ (question #4)**