ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2018 SESSION

Open-book examination
Calculators : only authorized models
Duration : 3 hours

14-PH-A7 OPTICS

1. (25 points) Geometrical optics. Reflective prism.

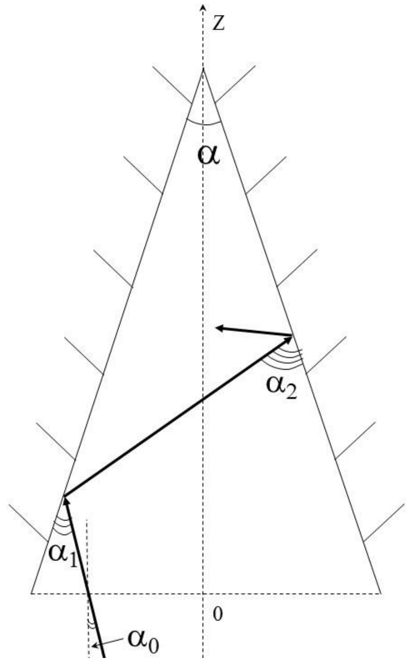

Fig. 1

In Fig. 1 a reflective prism is shown. The prism is made of two flat perfectly reflective mirrors put together at an angle α, with OZ being the prism axis of symmetry. A ray of light is incident onto the prism at an angle α_0. If the prism angle is small enough, the ray will continue moving forward along the OZ axis even after several reflections by the prism sides (see Fig. 1).

a. (10 points) Assuming that after *n-1* reflections by the prism sides the light still moves in the forward direction along the OZ axis, find an incident angle α_n of the light ray onto the prism side for the n^{th} reflection.
b. (10 points) Find the maximum number of reflections of the light ray by the prism sides after which the light is turned back and starts propagating in the backward direction along the OZ axis.
c. (5 points) Assuming a normal angle of incidence onto the prism α_0=0, what is the condition for the prism angle α so that only after one reflection by the prism side, the ray is turned and stars propagating backward along the OZ axis.

2. (25 points) Wave optics. Multiple reflections in the semi-spherical lens.

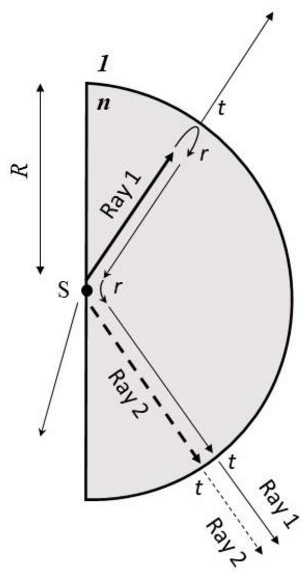

Fig.2

In Fig. 2 we show a semi-spherical lens of radius R and refractive index n placed in air with refractive index 1. In the center of a semi-spherical lens, a broadband point source S of coherent radiation is placed that emits at all angles. When tracking the paths of the individual rays, one observes that some rays (say ray 1 in Fig. 2) emitted by the point source S, can experience multiple reflections at the lens/air boundaries. This results in the interference of the direct ray (ray 2 in Fig. 2) and multiply reflected rays (ray 1 in Fig. 2, as well as others) coming out of the lens at the same angle. As a result, intensity if the outgoing beam will show spectral maxima or minima corresponding to the conditions of constructive or destructive interference of light inside of the lens.

a. (10 points) By summing up all the contributions from the direct and multiply reflected rays (in the lens), find the resultant electric field of the outgoing beam:
$$E = E_0 \cdot t \cdot exp(i \cdot Rk_l) + E_0 \cdot t \cdot r^2 \cdot exp(i \cdot 3Rk_l) + \cdots ,$$
where r and t are the Fresnel reflection and transmission coefficients at the lens/air boundary, k_l is a light propagation constant (wavenumber) in the lens material, and E_0 is the field amplitude of rays emitted by the source. For simplicity, we consider that Fresnel reflection coefficient r is angle independent at all the lens/air boundaries.

b. (10 points) Find an expression for the frequency dependent intensity of the outgoing beam:
$$I(\nu) = E(\nu) \cdot E^*(\nu).$$
Find expressions (in terms of the reflection and transmission coefficients) for the maximal $I_{max} = \max(I(\nu))$ and minimal $I_{min} = \min(I(\nu))$ values of the outgoing beam intensities, as well as corresponding frequencies ν_{max}, ν_{min} at which such values are achieved.

c. (5 points) Taking the following expressions for the Fresnel reflection and transmission coefficients at the lens/air boundaries:
$$t = \frac{2n}{n+1}\ ; r = \frac{n-1}{n+1}$$
calculate the spectral modulation intensity of the outgoing beam I_{max}/I_{min} in terms of the lens and air refractive indices.

3. (25 points) Wave optics. Diffraction on a slit.

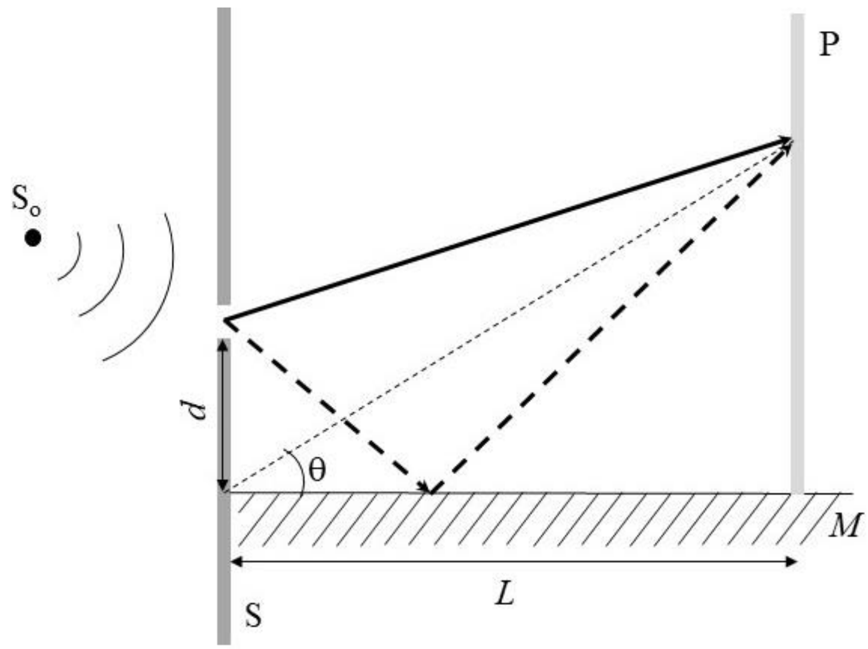

Fig. 3

Consider an opaque screen S featuring a single narrow infinite slit (see Fig. 3). The screen is illuminated from the left by a monochromatic point source So of wavelength λ. To the right of the screen S, there is a semi-infinite mirror M placed parallel to the slit and perpendicular to the screen S. The mirror is at a distance d from the slit.

a. (10 points) Describe an intensity distribution pattern $I(\theta)$ formed at the observation screen P which is positioned at a distance $L >> d$ from the screen S (screen P is parallel to the screen S).
b. (10 points) What are the angular positions $\theta_{max}, \theta_{min}$ of the intensity maxima $\max(I(\theta))$ and intensity minima $\min(I(\theta))$?
c. (5 points) Will the interference pattern change if the spatial position of a point source So is changed. If yes, then how?

4. (25 points) Wave optics. Divergence of the aperture-limited beam.

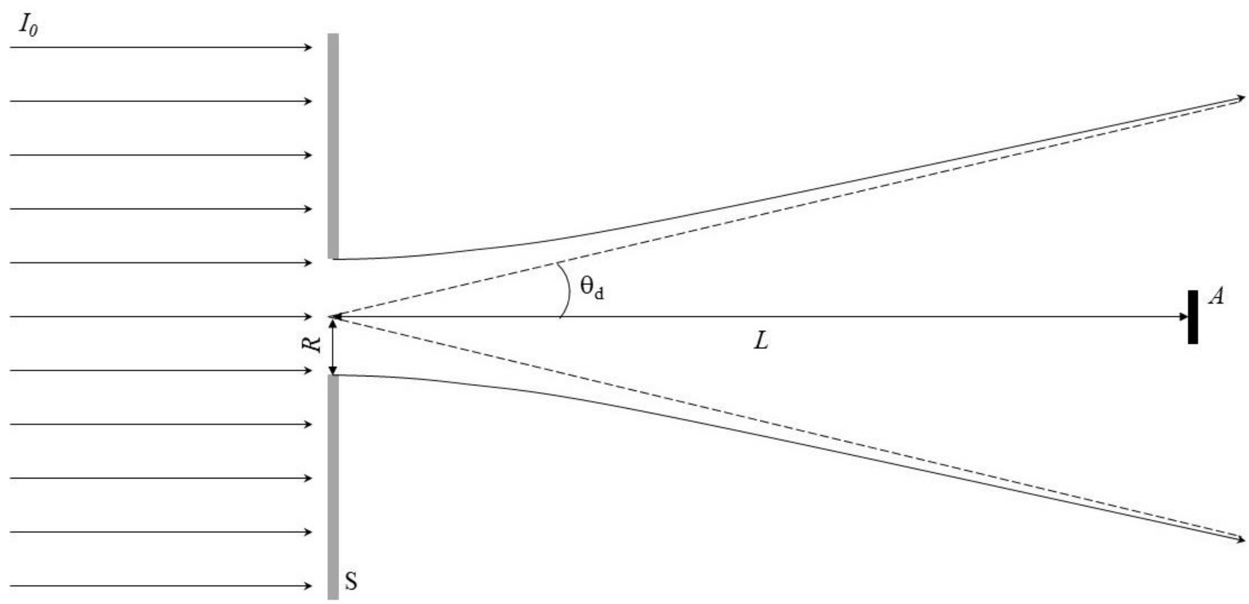

Fig. 4

In Fig. 4, monochromatic parallel beam of wavelength λ is incident on the circular opening of radius R cut in the infinite opaque screen S. This model can be used to describe behavior of a laser beam coming out of the finite-size aperture in the laser casing. As a result of diffraction on the opening, laser beam shows divergence with a characteristic angle θ_d.

a. (5 points) Give an expression for the beam divergence angle θ_d as a function of λ and R.
b. (15 points) Assuming that the light intensity of the parallel beam is I_0 *[W/m²]*, find an approximate value of the total power (in *[W]*) measured by the light detector with area A placed at a distance L from the opening as shown in Fig. 4 (assume a far field limit $L >> R^2/\lambda$)?
c. (5 points) Assuming that, in addition to diffraction, propagating beam is also absorbed in the air with an absorption coefficient (by power) α *[1/m]*, what would be the total power measured by the same detector as described in b.