ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2015 SESSION

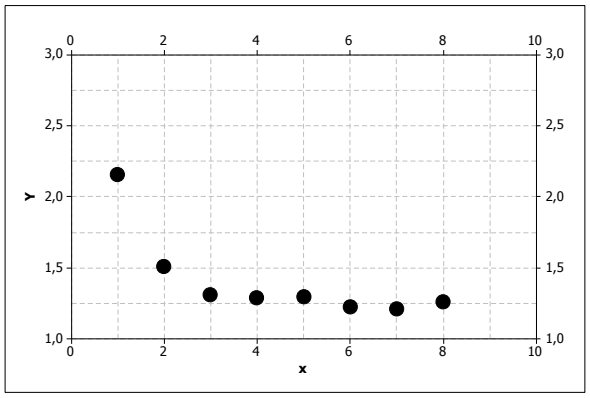
Toute documentation permise / Documentation are permited
Calculatrices : modèles autorisés seulement / Calculators: models allowed only
Durée de l'examen : 3 heures/ 3 hours

14-IN-A7 Applied Probability and Statistics

Question n° 1 (30 points)

It seeks to identify an empirical relationship $Y = f(X)$. A total of eight ($n = 8$) experiments were carried out for different values of X and Y (see table and graph).

X	Y	X	Y
1.00	2,155	5.00	1,293
2.00	1,507	6.00	1,219
3.00	1,307	7.00	1,204
4.00	1,284	8.00	1,253



a) (10 points) Considering the empirical model :

$$Y' = a_0 + \frac{a_1}{1+X} + \varepsilon$$

Where ε is a Gaussian noise with zero mean. You are asked to estimate the parameters a_0 and a_1 with the least square method.

b) (5 points)) Considering the following change of variable :

$$V = \frac{1}{1+X}$$

You are asked to estimate the linear correlation coefficient (Pearson coefficient) between and V and Y'.

c) (5 points) Can it be considered "significant"? Justify your answer by considering the sample size $n = 8$ and an error $\alpha = 0.05$.

d) (5 points) Is the value of a_0 significant or can it be considered "virtually" equal to zero? Justify your answer.

e) (5 points) If we consider a new variable $e = Y - Y'$, you are asked to perform hypothesis testing:

$$H_0 : \mu_e = 0$$

$$H_A : \mu_e \neq 0$$

Question n° 2 (10 points)

Consider X as a random variable which can be considered normal with average equal to 0 and a variance equal to 0.01 rad^2.

$$X \sim N\left(\mu_X = 0, \sigma_X^2 = 0.01\right)$$

Y is a new variable defined as:

$$Y = 2\left(X + 10\right)$$

a) (6 points) Calculate the mean and variance of the new variable Y.

b) (4 points) What is the distribution (pdf: probability density function) of the new variable Y? Give the mathematical expression and the new range for Y values.

Question n° 3 (20 points)

A research is conducted to demonstrate a relation between **exposure to chrysotile asbestos** and the **respiratory diseases**. A statistical study was conducted on **840** peoples. The results were classified into 3 categories: **Never exposed**, **occasionally exposed** and **continuously exposed**. (See the table):

Profile	Absence of pathology	Presence of pathology	**Total**
Never exposed	545	12	**557**
Occasionally exposed	154	19	**173**
Continuously exposed	75	35	**110**
Total	**774**	**66**	**840**

a) (5 points) With the assumption that the rate of respiratory diseases is related to exposure to chrysotile asbestos. Formulate statistical hypotheses H_0 and H_a for this contingency table.

b) (8 points) Can be regarded as plausible the hypothesis that exposure to chrysotile asbestos is a risk for respiratory diseases. Justify your answer. Use a 5% threshold for error TYPE I.

c) (7 points) Is there a significant difference between having '**occasional exposed**' or '**continuously exposed**'? Justify your answer.

Question n° 4 (20 points)

A batch contains 30 components including 10 **Class A** and 20 **Class B**. Three components $n=3$ are picked (randomly and without replacement).

a) (4 points) Calculate the probability of two components **Class A** in this batch.

b) (6 points) If the first component is not a **Class A**, what is the probability that the second and third picked components (randomly and without replacement) that both are **Class B**?

c) (5 points) Calculate the probability of getting 5 **Class B** components in a lot size of five ($n=5$).

d) (5 points) Calculate the probability of getting 2 components **Class A** AND 3 components **Class B** in a lot with the same size ($n=5$).

Question n° 5 (20 points)

Consider X as a continuous random variable with a distribution $f_X(x)$ with only one parameter ($\lambda \geq 0$):

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 5 \\ 0 & x < 5 \end{cases}$$

To estimate the value of $\lambda \geq 0$ a sampling was produced. Table below contains $n=20$ values.

5,95	5,31	5,75	5,48	6,75
6,37	5,05	5,02	6,70	5,59
6,17	6,29	5,15	5,14	5,75
5,50	5,83	5,03	7,27	5,54

a) (6 points) Using the method of moments, what is the best value of $\lambda \geq 0$.

b) (4 points) Calculate the expected value of the variable X.

c) (2 points) Calculate the probability $\Pr(x \geq 4)$.

d) (2 points) Calculate the probability $\Pr(x \leq 3)$.

e) (2 points) Calculate the probability $\Pr(x > 2\alpha | x \geq \alpha)$.

f) (2 points) Calculate the probability $\Pr(x = 6)$.

g) (2 points) Calculate the probability $\Pr(x \geq 6)$.

END