ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2018 SESSION

Toute documentation premise / Documentation are permited
Calculatrices : modèles autorisés seulement / Calculators: models allowed only
Durée de l'examen : 3 heures/ 3 hours

14-IN-A7 Applied Probability and Statistics

Question n° 1 (20 points)

X is an exponential continuous random variable with a PDF $f_X(x)$. A sampling was produced. Table below contains $n = 21$ values.

123,6	*40,8*	*18,2*	*63,0*	*137,2*	*57,6*	*59,1*
35,7	*52,5*	*2,3*	*115,1*	*43,7*	*63,4*	*60,8*
12,5	*193,9*	*2,8*	*100,3*	*57,2*	*91,9*	*187,3*

a) (5 points) Calculate the expected value of X : ($E\{X\}$).

b) (5 points) Using the method of moments to estimate the best parameter $\lambda \geq 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

c) (3 points) Calculate the probability of $\Pr\{X \leq 100\}$?

d) (3 points) Calculate the probability of $\Pr\{X = 50\}$?

e) (4 points) Calculate the probability $\Pr\{125 \leq X \leq 150\}$?

Question n° 2 (20 points)

An experiment has established that there is a correlation between the *Power* P [Watt] and the electrical current I [Amp]. The figure illustrates the set of experimental data. The values of the experimental measurements are given in the table below (the measurement error is estimated at ± 25 W for the power and ± 0.05 Amp for a 95% confidence level).

I (Amp)	53	54	61,5	63	70,5	71	72	74	77	79	79	80
P (Watt)	33731	34155	45428	48034	60171	61790	61652	66641	70363	74770	74045	76561

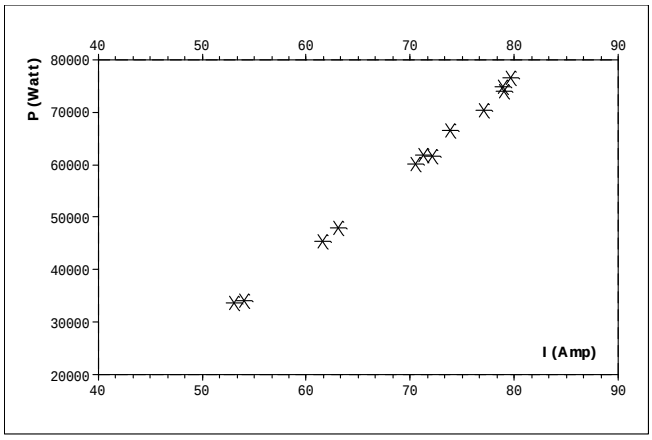

Figure 1. Relationship Power-Current (question 2)

You are asked :

a) (10 points) Considering the model $P = a_1 I^2 + \varepsilon$, where ε is a Gaussian random variable $N\left(0, \sigma^2\right)$. Estimate the parameter a_1 with the least square method.

b) (3 points) Is the value of a_1 is statistically significant? Justify your answer by a statistical test.

c) (7 points) Calculate the linear correlation coefficient (Pearson's coefficient) between variables P and I (attention : not I^2). Is coefficient can be considered as 'significant'? Justify your answer considering the sample size $n = 12$ and Alpha tolerated error $\alpha = 0.05$.

Question n° 3 (15 points)

Let X be a Gaussian random variable with expectation equal to 3 and a unit variance $X = N\left(\mu_x = 3, \sigma_x^2 = 1\right)$. Y is a new variable defined as :

$$Y = \sum_{i=1}^{n} X_i$$

a) (8 points) Calculate the expected value μ_y and variance σ_y^2 of the new variable Y.

b) (7 points) A variable change was made, such as: : $Z = \left(X - 3\right)$, you are asked to calculate the expectation μ_u and variance σ_u^2 of the new variable :

$$U = \frac{1}{n} \sum_{i=1}^{n} Z_i$$

Question n° 4 (15 points)

The behaviour of *waiting time* of a transaction is modelled by an exponential distribution law with an expected value of 2 minutes.

a) (3 points) Give the mathematical expression of the probability density function (pdf) of the random variable: *Waiting time*.

b) (4 points) Determine the probability that the delay is equal to 2 minutes.

c) (4 points) Determine the probability that the delay is less than 2 minutes.

d) (4 points) Determine the probability that the delay is greater than 6 minutes.

Question n° 5 (10 points)

In a batch of 45 components, five (5) are no conforms. We pick randomly and without replacement, 3 components among the 45. We ask you:

a) (4 points) Estimate the probability of 2 no conform components.

b) (3 points) What is the probability that the second component is no conform?

c) (3 points) Considering that the second component was no conform, what is the probability that the first component was not?

Question n° 6 (15 points)

Suppose that there are two manufacturing methods (A and B) for a product and that both methods generate ***Type 1*** and ***Type 2*** defects. We want to know if there is a significant relationship between the method used and the type of defects met. The data is summarized in a contingency table.

	Method **A**	Method **B**	
Defect Type #1	**20**	**35**	Total = 55
Defect Type #2	**20**	**15**	Total = 35

a) (5 points) Formulate the statistical hypotheses H_0 and H_a for this contingency table.

b) (10 points) Can we consider as likely the hypothesis that the method employed has no influence on the type of defect? Justify your answer by a statistical test.

Question n° 7 (10 points)

You must estimate the performance of a supplier. A measurement campaign was conducted and the following statistics were obtained:

	Supplier
Sample Size *n*	125
Estimated mean	49.7
Estimated standard deviation	2.60
Maximum value	53.8
Minimum value	46.3

You are asked :

a) (10 points) If the requirement is **50±5**, what is the percentage of nonconformity of this supplier?

END