ORDRE DES INGÉNIEURS DU QUÉBEC

NOVEMBER 2015 SESSION

Toute documentation permise / Documentation are permited
Calculatrices : modèles autorisés seulement / Calculators: models allowed only
Durée de l'examen : 3 heures/ 3 hours

14-IN-A7 Applied Probability and Statistics

Question n° 1 (10 points)

The waiting time X (in minutes) to obtain IT support to the Ministry of Finance is a random variable (RV) that follows an exponential distribution law with expected value $E\{X\}$ equal to 10 minutes.

a) (4 points) Determine the mathematical expression of the probability density function PDF governing the RV waiting time.
b) (2 points) Calculate the probability that the waiting time is equal to 10 minutes.
c) (2 points) Calculate the probability that the waiting time is less than 3 minutes.
d) (2 points) Calculate the probability that the delay is greater than 2 times the expected value $E\{X\}$.

Question n° 2 (15 points)

The addition of an additive (color pigment) in a plastic resin has in theory no effect on its resistance to UV light. A total of 1750 tests were tested. A sample of 750 individuals was deemed conform and 1000 individuals were reported as non-conform. The additive was added to 969 individuals (781 individuals tested had no additive).

Total	*With additive*	*Without additive*
Conform (750)	*419*	*331*
Non-conform (1000)	*550*	*450*

a) (5 points) Formulate statistical hypotheses H and Ha for this contingency table.
b) (10 points) Can we consider as probable the hypothesis that the addition of additive in the plastic resin modifies significantly its resistance to UV?

Question n° 3 (20 points)

A supplier sends you a batch of 100,000 pills. You want to control weight (milligrams). The functional requirement is 50 ± 2 mg. With a random sample equal to 500 pills, you estimated the average 49,750 ± 0,027 mg and standard deviation of the sample 0,750 ± 0,017 mg. Considering a Gaussian distribution for the weight, you are asked:

a) (4 points) Calculate the probability that the weight is less than 48 mg.

b) (4 points) Calculate the probability that the weight is greater than 52 mg.

c) (4 points) Calculate the probability that the weight is within the tolerance (50 ± 2 mg).

d) (4 points) Determine the number of non-conform pills that you expect on a lot of one million pills (DPMO).

e) (4 point) What is the mathematical expression of the probability density function (PDF) governing the weight of 100 pills?

Question n° 4 (20 points)

LED provider guarantees a minimum operating life of 15,000 hours (with a risk of 5%). You tested a sample of 15 LED until they broke. The results of their lifetimes (in hours) are the following:

21979	11178	22524	24005	27620	25346
24456	19596	20322	39720	56213	32782
55943	13085	14998			

a) (5 points) Calculate the expected life of LED and its 95% confidence interval.

b) (5 points) For more accuracy in predicting the lifetime (maximum error = ± 1000 hours), what is the minimum size of the test sample you need to consider?

c) (10 points) Can you confirm that the supplier respects the 15,000 hours threshold of 95%? Consider a distribution type:

$$f_X(x) = \begin{cases} \dfrac{1}{E\{x\}} e^{-\frac{(x-10000)}{E\{x\}}} & x > 10000 \\ 0 & x \leq 10000 \end{cases}$$

Question n° 5 (20 points)

The probability density function (PDF) of a continuous random variable X (with $X \geq x_0$) is given by:

$$f_X(x) = \begin{cases} \alpha e^{-\alpha(x-x_0)} & x > x_0 \\ 0 & x \leq x_0 \end{cases}$$

With $\alpha > 0$ and $x_0 > 0$.

a) (5 points) Calculate $P[X = x_0] + P[X > 2x_0]$

b) (5 points) Calculate $P[X > 3x_0 | X \geq x_0]$

c) (5 points) What is the statistical expectation of the random variable $Y = |X| + \pi$?

d) (5 points) If a new random variable is defined: $Z = X - x_0$, calculate the PDF $f_Z(z)$.

Question n° 6 (15 points)

A linear correlation study between the **X** and **Y** was performed on an unbiased sample. The results are shown in Table below.

X	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Y	*10.00*	*10,32*	*11,16*	*11,48*	*11,79*	*11,96*	*12,12*	*12,21*

a) (10 points) Considering the model $\mathbf{Y} = a_1\mathbf{X} + a_0 + \varepsilon$ (where $\varepsilon = N(0, \sigma^2)$). Estimate the parameters a_0 and a_1 with least squares method.

b) (5 points) Comment on the quality of this model ($\mathbf{Y} = a_1\mathbf{X} + a_0$). Is it a good model? Why?

End