ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2018 SESSION

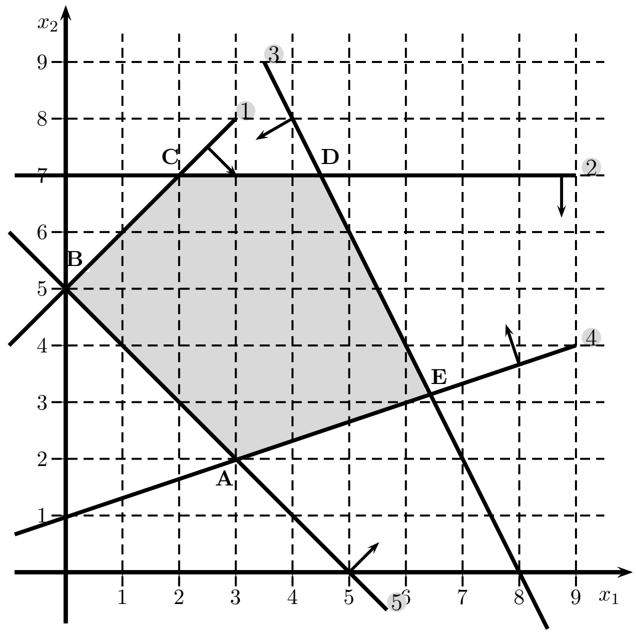
Open-book examination
Calculators : only authorized models
Duration : 3 hours

14-IN-A1 OPERATIONS RESEARCH

NOTE :
PLEASE ANSWER DIRECTLY
ON THE QUESTIONNAIRE

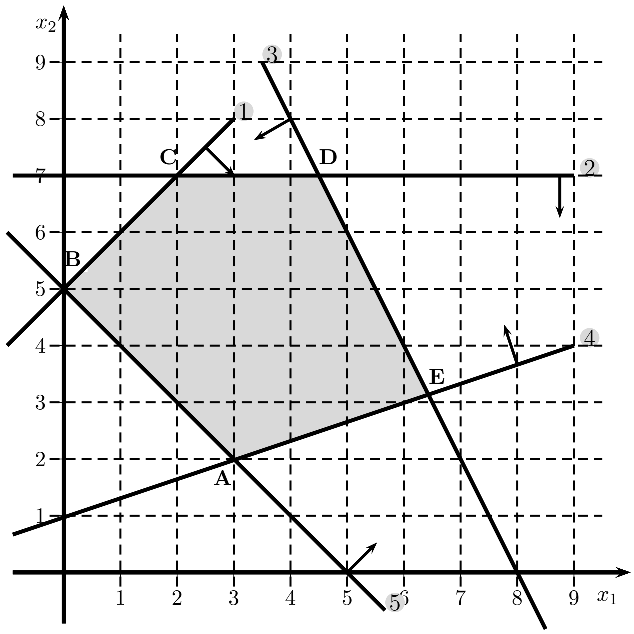
Question no 1 (18 points)

The shaded area of the following graph represents the feasible region of a linear programming problem.



(a) (10 points)

Give all the constraints associated with this feasible region.



(b) (4 points)

If the objective function is max $Z = 3x_1 + x_2$, find the optimal solution and give the exact coordinates of the corner-point associated with this optimal solution.

(c) (4 points)

If the objective function is min $Z = c_1x_1 + 3x_2$, which value(s) should be given to c_1 in order to find an optimal solution at point A in the figure ? Justify your answer.

Question no 2 (16 points)

A company can machine gears for bicycles on 5 different machines. The following table summarizes the production costs associated with the machining of the gears on the different machines, as well as the time in minutes to machine a gear on each one. The fixed cost only applies if the machine is turned on for production. Unit costs represent, for each machine, the manufacturing cost for one gear. When a machine is used, it must manufacture at least 200 units.

Machine	Fixed Cost ($)	Unit Costs ($)	Mchining Time (minutes)
1	825	26	0,5
2	750	24	0,4
3	900	22	0,5
4	1022	23	0,6
5	825	21	1,2

(a) (8 points)

During the next 8-hour shift, the company must manufacture 2,000 gears. You are asked to build a linear model that will determine how it should distribute manufacturing between machines to minimize total production costs. Clearly define your variables.

(b) (4 points)

Given the air quality standards, if machine 1 is running, then at most 2 other machines can operate. Modify the model presented in (a) to reflect this new constraint.

(c) (4 points)

Remark: *This sub-question is independent of sub-questions (a) and (b).*

Let y_A, y_B, y_C and y_D be four binary variables representing four events called A, B, C and D. If event A occurs and event B does not occur, then event C and event D must occur. Using the binary variables, indicate one or more constraints that would allow to model this situation.

Question no 3 (14 points)

Consider the following linear program:

$$
\begin{array}{lcl}
\max Z = x_1 - x_2 + 2x_3 & & \\
\text{subject to:} & & \\
-4x_1 + x_2 + \ \ x_3 & \leq & 20 \\
\ \ x_1 \qquad\quad + 3x_3 & \leq & 60 \\
2x_1 - x_2 + \ \ x_3 & \leq & 10 \\
x_1, x_2, x_3 & \geq & 0
\end{array}
$$

(a) (12 points)

Solve this problem using the simplex algorithm.

Basic Variables									Value

Basic Variables									Value

Basic Variables									Value

Basic Variables									Value

(b) (2 points)

Does this linear program have more than one optimal solution? Justify your answer.

Question no 4 (20 points)

Consider the following linear program

$$
\begin{aligned}
&\max\ Z = 20x_1 + 60x_2 + 140x_3 \\
&\text{subject to:} \\
&12x_1 + 20x_2 + 15x_3 \leq 200 \quad (1) \\
&3x_1 + 4x_2 + 2x_3 \geq 20 \quad (2) \\
&x_1 - x_3 \leq 10 \quad (3) \\
&x_2 \geq 4 \quad (4) \\
&x_1 + x_2 \leq 18 \quad (5) \\
&x_1,\ x_2,\ x_3 \geq 0
\end{aligned}
$$

The optimal solution of the linear program is presented in the following Simplex Tableau. The variables e_1, e_2, e_3, e_4 and e_5 represent the slack and the excess variables used in each constraint and the variable a_2 and a_4 are the artificial variables used in constraints 2 and 4.

Basic Variables	Z	x_1	x_2	x_3	e_1	e_2	e_3	e_4	e_5	a_2	a_4	Value
Z	1	92	0	0	9,33	0	0	126,67	0	M	M-126,67	1360
e_2	0	-1,4	0	0	0,13	1	0	-1,33	0	-1	1,33	12
x_3	0	0,8	0	1	0,07	0	0	1,33	0	0	-1,33	8
e_3	0	1,8	0	0	0,07	0	1	1,33	0	0	-1,33	18
x_2	0	0	1	0	0	0	0	-1	0	0	1	4
e_5	0	1	1	0	0	0	0	1	1	0	-1	14

(a) (5 points)

Write the dual formulation of this problem.

$$\begin{aligned} \max\ Z &= 20x_1 + 60x_2 + 140x_3 \\ \text{subject to:} & \\ 12x_1 + 20x_2 + 15x_3 &\leq 200 \quad (1) \\ 3x_1 + 4x_2 + 2x_3 &\geq 20 \quad (2) \\ x_1 - x_3 &\leq 10 \quad (3) \\ x_2 &\geq 4 \quad (4) \\ x_1 + x_2 &\leq 18 \quad (5) \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

Basic Variables	Z	x_1	x_2	x_3	e_1	e_2	e_3	e_4	e_5	a_2	a_4	Value
Z	1	92	0	0	9,33	0	0	126,67	0	M	M-126,67	1360
e_2	0	-1,4	0	0	0,13	1	0	-1,33	0	-1	1,33	12
x_3	0	0,8	0	1	0,07	0	0	1,33	0	0	-1,33	8
e_3	0	1,8	0	0	0,07	0	1	1,33	0	0	-1,33	18
x_2	0	0	1	0	0	0	0	-1	0	0	1	4
e_5	0	1	1	0	0	0	0	1	1	0	-1	14

(b) (5 points)

Based on the Simplex Tableau of the optimal primal solution, give the optimal solution of the dual; i.e. the value of the variables as well as the value of the objective function. Justify your approach.

(c) (5 points)

For which values of the coefficient of x_3 in the objective function (i.e. $c_3 = 140$) does the current solution remain optimal? Justify your answer.

max $Z = 20x_1 + 60x_2 + 140x_3$
subject to:

$$\begin{array}{rcll} 12x_1 + 20x_2 + 15x_3 & \leq & 200 & (1) \\ 3x_1 + 4x_2 + 2x_3 & \geq & 20 & (2) \\ x_1 - x_3 & \leq & 10 & (3) \\ x_2 & \geq & 4 & (4) \\ x_1 + x_2 & \leq & 18 & (5) \\ x_1,\ x_2,\ x_3 & \geq & 0 & \end{array}$$

Basic Variables	Z	x_1	x_2	x_3	e_1	e_2	e_3	e_4	e_5	a_2	a_4	Value
Z	1	92	0	0	9,33	0	0	126,67	0	M	M-126,67	1360
e_2	0	-1,4	0	0	0,13	1	0	-1,33	0	-1	1,33	12
x_3	0	0,8	0	1	0,07	0	0	1,33	0	0	-1,33	8
e_3	0	1,8	0	0	0,07	0	1	1,33	0	0	-1,33	18
x_2	0	0	1	0	0	0	0	-1	0	0	1	4
e_5	0	1	1	0	0	0	0	1	1	0	-1	14

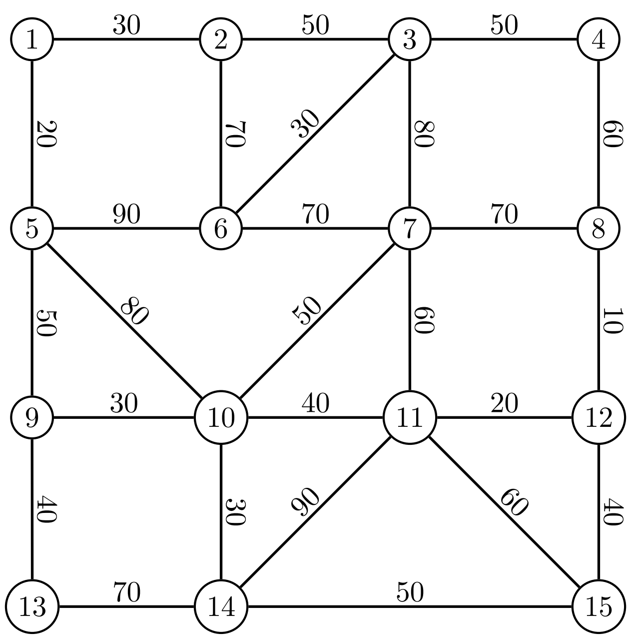
(d) (5 points)

If the right-hand side of the fourth constraint ($b_4 = 4$) increased by 3 units (i.e. $b_4 = 7$), what will be the impact on the value of variables as well as the value of Z?

Question no 5 (12 points)

Fifteen motion detectors have just been installed in a section of a museum (nodes 1 to 15 on the following graph). These detectors will be activated when the museum is closed. As soon as a movement is detected by a detector, a signal must be sent the other detectors.

It is desired to carry out the cabling in order to connect the motion detectors with each other. The edges of the following graph represent potential links between certain motion detectors and the number above the edges indicate the length of each link in meters. We seek the subset of the edges of this network that will minimize the total cable length that will allow communication between all pairs of motion detectors. Indicate the optimal configuration on the following graph as well as the total length.



Question no 6 (20 points)

The following table is associated with a transport problem between four suppliers (F_1, F_2, F_3 et F_4) and three customers (C_1, C_2 et C_3). The numbers in the shaded areas represent the unit transportation costs between a supplier and a customer.

(a) (6 points)

Find a feasible solution using Vogel's approximation method.

	C_1	C_2	C_3	Offer
F_1	17	22	11	40
F_2	30	11	19	40
F_3	10	11	6	30
F_4	18	20	15	40
Demand	60	30	60	150

(b) (12 points)

From the starting solution obtained in (a), find the optimal solution.

	C_1	C_2	C_3	Offer
F_1	17	22	11	40
F_2	30	11	19	40
F_3	10	11	6	30
F_4	18	20	15	40
Demand	60	30	60	150

	C_1	C_2	C_3	Offer
F_1	17	22	11	40
F_2	30	11	19	40
F_3	10	11	6	30
F_4	18	20	15	40
Demand	60	30	60	150

	C_1	C_2	C_3	Offer
F_1	17	22	11	40
F_2	30	11	19	40
F_3	10	11	6	30
F_4	18	20	15	40
Demand	60	30	60	150

(c) (2 points)

Is the solution found in (b) the unique optimal solution? Justify your answer.