ORDRE DES INGÉNIEURS DU QUÉBEC

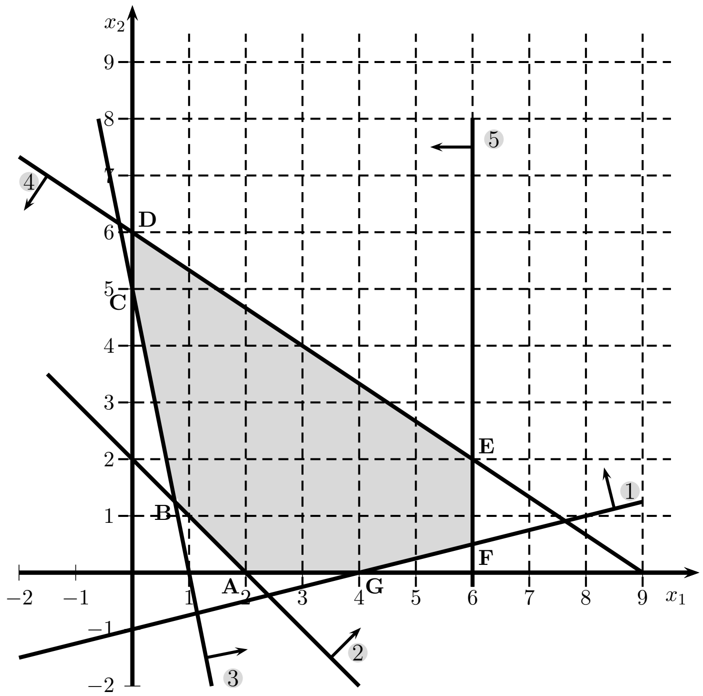
NOVEMBER 2020 SESSION

Open-book examination
Calculators : only authorized models
Duration : 3 hours

14-IN-A1 OPERATIONS RESEARCH

Question no 1 (20 points)

The shaded area of the following graph represents the feasible region of a linear programming problem.



(a) (12 points)

Give all the constraints associated with this feasible region.

(b) (4 points)

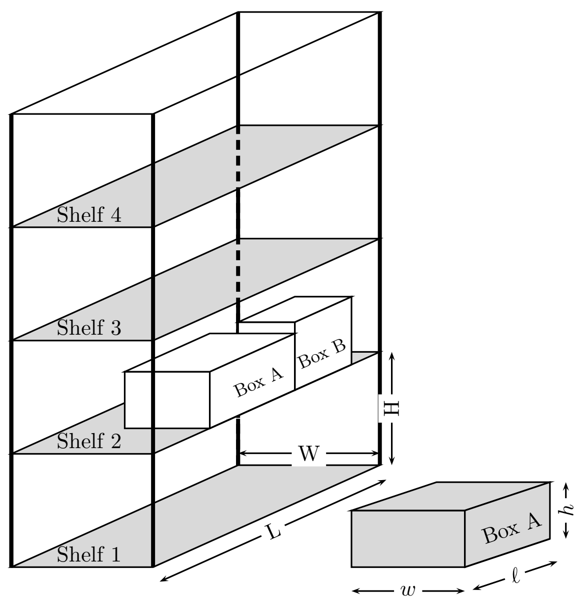
If the objective function is min $Z = 3x_1 + x_2$, find the optimal solution and give the exact coordinates of the corner-point associated with this optimal solution.

(c) (4 points)

If the objective function is max $Z = c_1x_1 + 3x_2$, which value(s) should be given to c_1 in order to find an optimal solution at point E in the figure ? Justify your answer.

Question no 2 (18 points)

We want to store several boxes on a rack that contains four shelves (numbered 1 to 4 in the following figure). The lenght ($L = 2000$ cm), the height($H = 80$ cm) and the width ($W = 100$ cm) of each shelf is shown on the figure. There are 4 type of boxes, noted A, B, C and D. The figure shows the only permitted position of the boxes on the shelves. No rotation is allowed. In addition, we cannot put one box on top of another, nor one box behind the other (i.e. we have to put the boxes side by side along the length of the rack as shown on the figure). The width (w) and height (h) of each type of boxes respect the width (W) and height (H) of the shelves in the rack, i.e. $h < H$ and $w < W$. The table on the right indicates the number of boxes of each type and their characteristics (length, noted ℓ, and weight).



Box	Number	Length (cm)	Weight (kg)
A	40	26	25
B	60	24	30
C	50	22	20
D	80	17	18

Shelves 2 to 4 can support a maximum weight of 400 kg.

(a) (10 points)

You are asked to make a linear program that will maximize the total number of boxes that can be stored on the shelves. Clearly define your variables.

(b) (4 points)

If at most three different types of boxes are allowed on each shelf (the types of boxes can vary from one shelf to another), how would you modify the model proposed in (a) to take account of this new constraint. Modify the model presented in (a) to reflect this new constraint.

(c) (4 points)

Remark: *This sub-question is independent of sub-questions (a) and (b).*

Let y_A, y_B, y_C and y_D be four binary variables representing four events called A, B, C and D. If events A and B occur, then event C must occur but not event D. Using the binary variables, indicate one or more constraints that would allow to model this situation.

Question no 3 (16 points)

Consider the following linear program:

$$\begin{array}{rcrcrcrcrcr}
\max Z & = & 6x_1 & + & 4x_2 & + & 3x_3 & + & 2x_4 & & \\
s.c. & & & & & & & & & & \\
& & & & 2x_2 & + & x_3 & - & 2x_4 & \geq & 2 \\
& & x_1 & + & x_2 & + & 2x_3 & & & \leq & 5 \\
& & x_1 & + & 2x_2 & & & - & x_4 & \leq & 10 \\
& & x_1, & & x_2, & & x_3, & & x_4 & \geq & 0
\end{array}$$

(a) (14 points)

Solve this problem using the simplex algorithm.

(b) (2 points)

Does this linear program have more than one optimal solution? Justify your answer.

Question no 4 (10 points)

Consider the following linear program

$$
\begin{array}{l}
\max\ Z = 3x_1 + 4x_2 + 2x_3 \\
\text{subject to:} \\
\begin{array}{rcrcrcrl}
x_1 & + & 5x_2 & - & 3x_3 & \geq & 26 & (1) \\
3x_1 & + & 8x_2 & + & 7x_3 & \leq & 100 & (2) \\
x_1 & + & x_2 & + & x_3 & = & 18 & (3) \\
6x_1 & & & + & 4x_3 & \leq & 80 & (4) \\
x_1 & , & x_2 & , & x_3 & \geq & 0 &
\end{array}
\end{array}
$$

The optimal solution of the linear program is presented in the following Simplex Tableau. The variables e_1, e_2 and e_4 represent the slack and the excess variables used in constraints 1,2 and 4 respectively; while the variables a_1 and a_3 are the artificial variables used in constraints 1 and 3, respectively.

Basic Variables	Z	x_1	x_2	x_3	e_1	a_1	e_2	a_3	e_4	Value
Z	1	0	0	0	-0,25	-M+0,25	0	-M+2,75	0	56
x_2	0	0	1	0	-0,111	0,111	0,111	-0,444	0	6
x_3	0	0	0	1	0,139	-0,139	0,111	-0,194	0	4
x_1	0	1	0	0	-0,028	0,028	-0,222	1,639	0	8
e_4	0	0	0	0	-0,389	0,389	0,889	-9,056	1	16

(a) (5 points)

Write the dual formulation of this problem.

max $Z = 3x_1 + 4x_2 + 2x_3$
subject to:

$$
\begin{array}{rcrcrcrl}
x_1 & + & 5x_2 & - & 3x_3 & \geq & 26 & (1) \\
3x_1 & + & 8x_2 & + & 7x_3 & \leq & 100 & (2) \\
x_1 & + & x_2 & + & x_3 & = & 18 & (3) \\
6x_1 & & & + & 4x_3 & \leq & 80 & (4) \\
x_1 & , & x_2 & , & x_3 & \geq & 0 &
\end{array}
$$

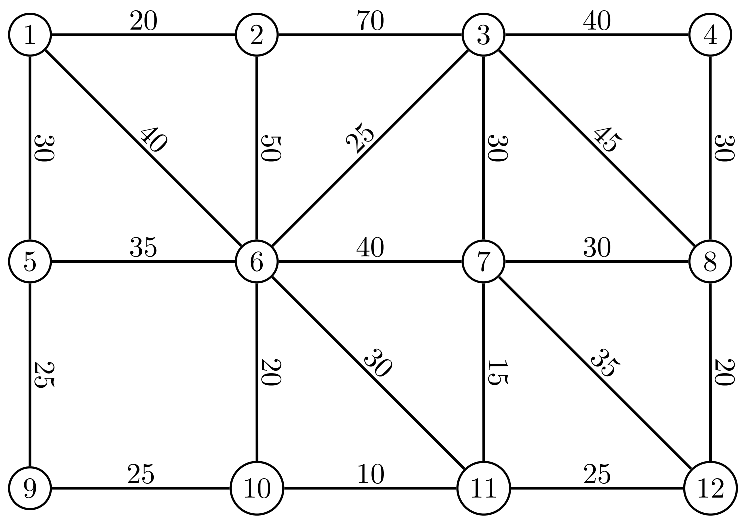
Basic Variables	Z	x_1	x_2	x_3	e_1	a_1	e_2	a_3	e_4	Value
Z	1	0	0	0	-0,25	-M+0,25	0	-M+2,75	0	56
x_2	0	0	1	0	-0,111	0,111	0,111	-0,444	0	6
x_3	0	0	0	1	0,139	-0,139	0,111	-0,194	0	4
x_1	0	1	0	0	-0,028	0,028	-0,222	1,639	0	8
e_4	0	0	0	0	-0,389	0,389	0,889	-9,056	1	16

(b) (5 points)

If the right-hand side of the second constraint ($b_2 = 100$) decreased by 9 units (i.e. $b_2 = 91$), what will be the impact on the value of variables as well as the value of Z? Justify your approach.

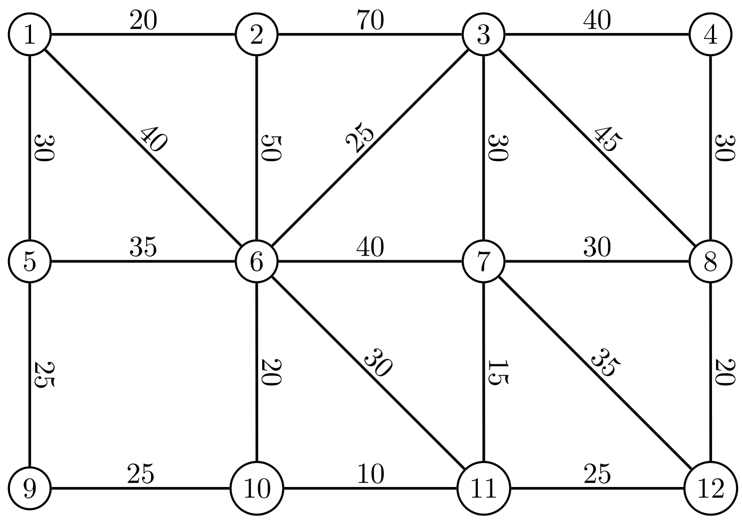
Question no 5 (10 points)

In the following graph, the nodes represent the buildings of a university campus. The edges represent the footpaths between the buildings and the value above each edge indicates the distance in meters between the buildings. The university wishes to install lampposts on certain segments to allow pedestrians to go from one building to another on a lighted path. Knowing that it costs about 50$/m to install the lighting, the university wants to know the segments that it must illuminate in order to minimize the costs of this project.



Question no 6 (10 points)

Find the shortest path between node 1 and the other nodes in the graph.



Question no 7 (16 points)

The following table is associated with a transport problem between three suppliers (S_1, S_2, and S_3) and four customers (C_1, C_2, C_3 and C_4). The numbers in the shaded areas represent the unit transportation costs between a supplier and a customer.

(a) (5 points)

Find a feasible solution using Vogel's approximation method.

	C_1	C_2	C_3	C_4	Offer
S_1	13	20	22	30	60
S_2	6	10	16	20	60
S_3	11	22	20	11	30
Demand	30	40	40	40	150

(b) (10 points)

Starting from the initial solution obtained in (a), find the optimal solution.

	C_1	C_2	C_3	C_4	Offer
S_1	13	20	22	30	60
S_2	6	10	16	20	60
S_3	11	22	20	11	30
Demand	30	40	40	40	150

	C_1	C_2	C_3	C_4	Offer
S_1	13	20	22	30	60
S_2	6	10	16	20	60
S_3	11	22	20	11	30
Demand	30	40	40	40	150

	C_1	C_2	C_3	C_4	Offer
S_1	13	20	22	30	60
S_2	6	10	16	20	60
S_3	11	22	20	11	30
Demand	30	40	40	40	150

(c) (1 point)

Is the solution found in (b) the unique optimal solution? Justify your answer.