ORDRE DES INGÉNIEURS DU QUÉBEC

NOVEMBER 2018 SESSION

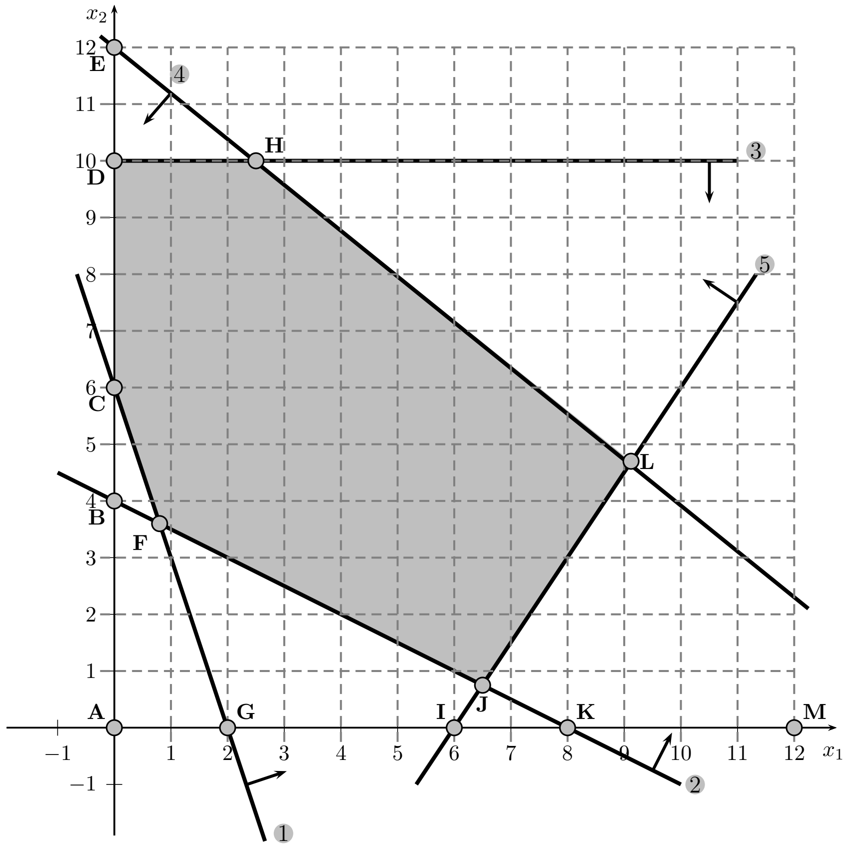
14-IN-A1

OPERATIONS RESEARCH

Open-book examination Calculators : only authorized models Duration : 3 hours **Answer on this exam booklet only**.

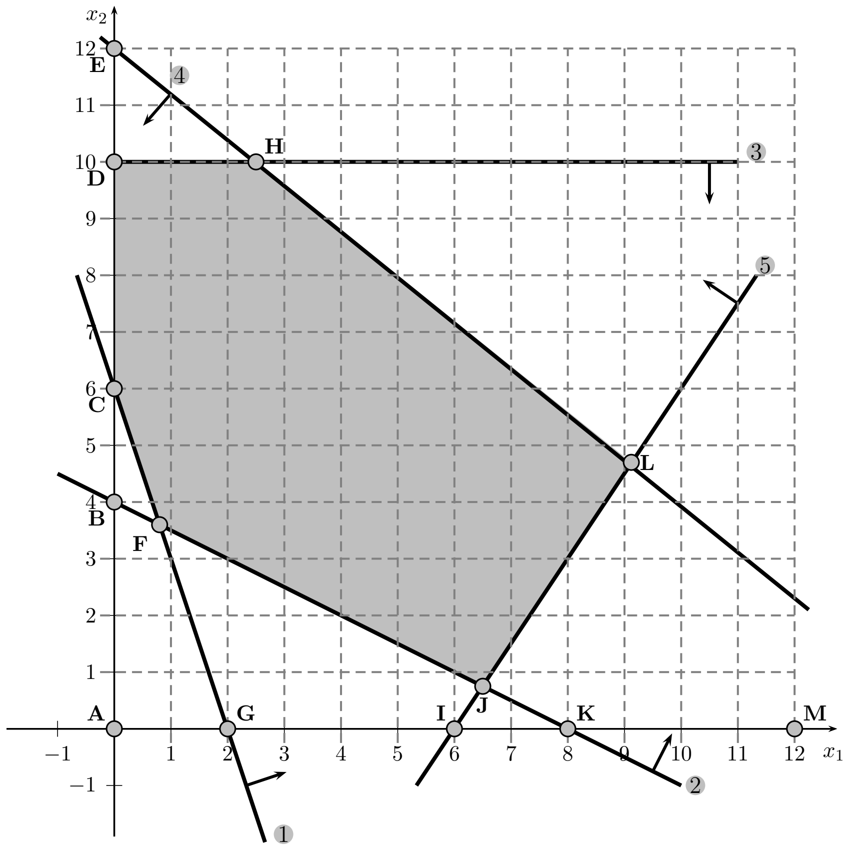
Question no 1 (20 points)

The shaded area of the following graph represents the feasible region of a linear programming problem.



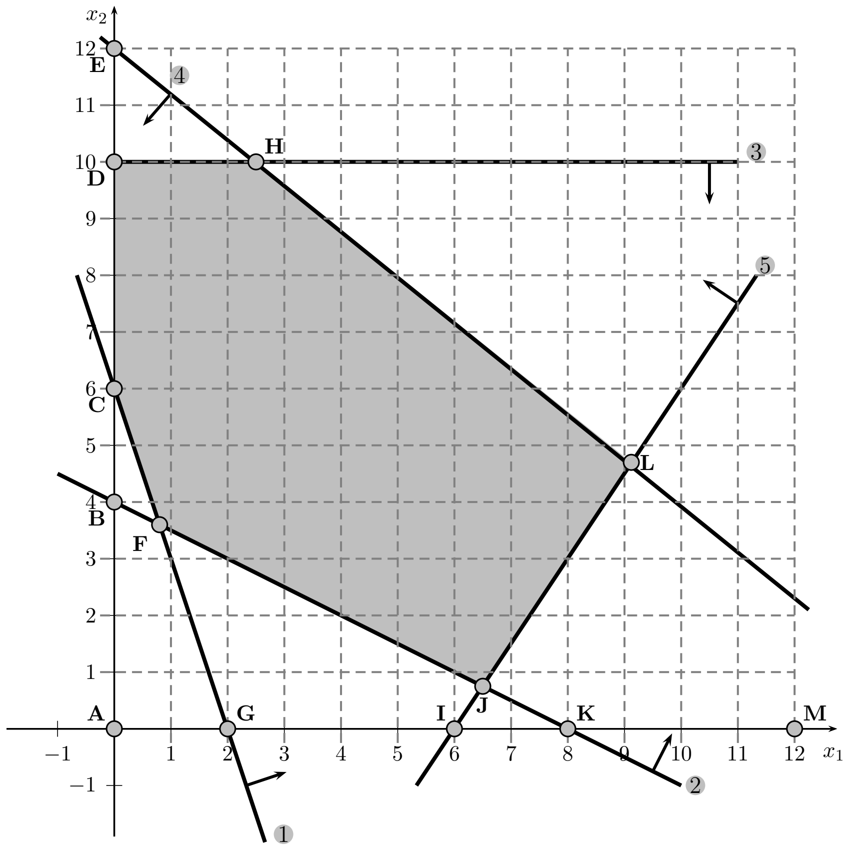
(a) (6 points)

Give all the constraints associated with this feasible region.



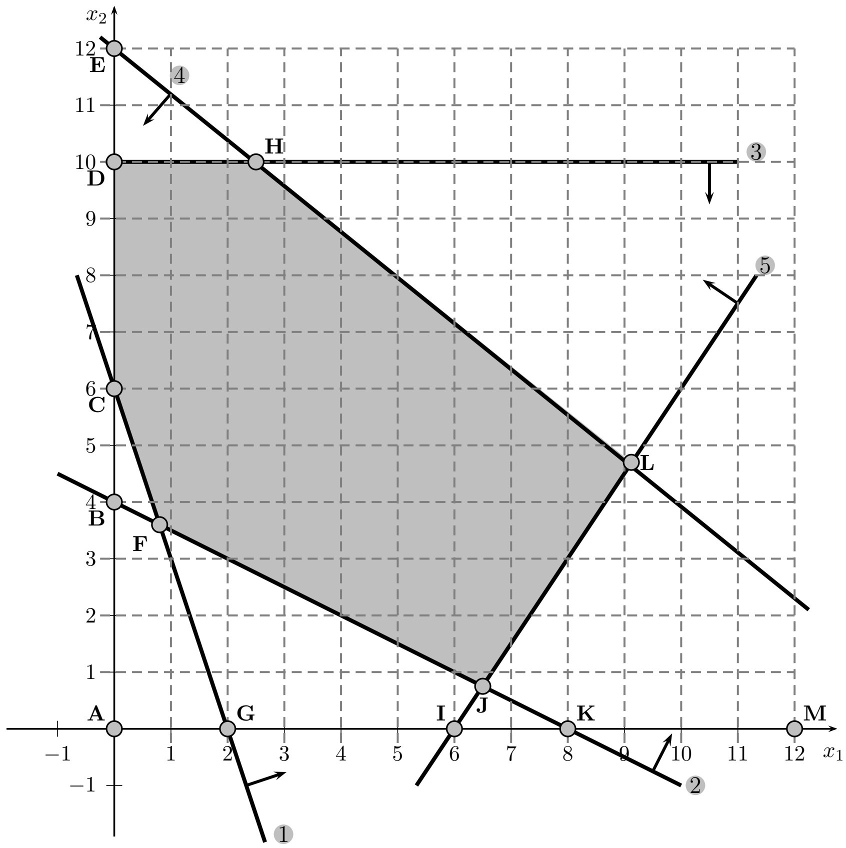
(b) (4 points)

If the objective function of this linear program is $\min Z = 2x_1 + x_2$, what is the optimal solution? Give the exact values of the variables and the value of Z using the equations of the binding constraints.



(c) (6 points)

If the objective function of this linear program is $\max Z = kx_1 + 2x_2$, what value(s) should we give to the parameter k so that the point L in the figure is an optimal and unique solution of the linear program?



(d) (4 points)

If the objective function of this linear program is $\max Z = 4x_1 - 5x_2$, and x_1 and x_2 must be integers, what is the optimal solution of the linear integer program ? Justify your answer.

Question no 2 (16 points)

A company makes 3 types of agricultural fertilizer: F_1, F_2, and F_3. Four chemicals, C_1, C_2, C_3 and C_4, are used to make fertilizers. Fertilizer F_1 must be composed of at least 50% of C_1. In addition, the sum of C_3 and C_4 must not exceed 20% of the total amount of F_1. In fertilizer F_2, the amount of C_2 must be at least 2 times greater than that of C_3. In addition, F_2 must contain at least 30% of C_2. Finally, in F_3 fertilizer, the amount of C_4 must be less than 20% of the total of C_1 and C_3.

The following table shows the cost ($/kg) of buying chemicals and the quantity (in kg) available.

Chemical	Cost ($/kg)	Quantity (kg)
C_1	6.25	10000
C_2	7.50	22000
C_3	9.00	15000
C_4	6.75	21500

The selling price of F_1, F_2 and F_3 fertilizers is 70$/kg, 72$/kg, and 80$/kg, respectively.

(a) (8 points)

Assuming that all fertilizer produced can be sold, formulate a linear programming model that can be used to maximize the company's profit. Clearly define your variables.

(b) (4 points)

If the C_2 and C_4 should never be mixed in the same fertilizer, how should the model shown in (a) be modified to account for this new constraint?

(c) (4 points)

Remark: *This sub-question is independent of sub-questions (a) and (b).*

Let y_A, y_B, y_C and y_D be four binary variables representing four events called A, B, C and D. If event A and event B occur, then event C and event D must occur. Using the binary variables, indicate one or more constraints that would allow to model this situation.

Question no 3 (14 points)

Consider the following linear program:

$$
\begin{array}{rcrcrcl}
\multicolumn{7}{l}{\max Z = -2x_1 + 3x_2 + 2x_3} \\
\multicolumn{7}{l}{\text{subject to:}} \\
 & & 2x_2 & + & x_3 & \leq & 120 \\
x_1 & - & x_2 & - & 4x_3 & \leq & 80 \\
-x_1 & + & x_2 & + & x_3 & \leq & 100 \\
 & & \multicolumn{3}{r}{x_1, x_2, x_3} & \geq & 0
\end{array}
$$

(a) (12 points)

Solve this problem using the simplex algorithm.

Basic Variables									Value

Basic Variables									Value

Basic Variables									Value

Basic Variables									Value

(b) (2 points)

Does this linear program have more than one optimal solution? Justify your answer.

Question no 4 (10 points)

We want to assign four jobs to four machines. The following table shows the manufacturing time in minutes if the job J_j is assigned to the machine M_i.

Jobs	Machines			
	J_1	J_2	J_3	J_4
M_1	12	9	14	17
M_2	11	10	15	15
M_3	13	11	15	18
M_4	11	12	13	16

(a) (8 points)

At which machine should we assign each job to minimize the total manufacturing time?

(b) (2 points)

Is this solution unique ? Justify your answer.

Question no 5 (20 points)

Consider the following linear program

$$
\begin{aligned}
&\max\ Z = 2x_1 - 2x_2 + 3x_3 \\
&\text{subject to:} \\
&\begin{array}{rcll}
x_1 - x_2 + x_3 & \leq & 100 & (1) \\
-4_1 + x_2 - x_3 & \leq & 80 & (2) \\
x_1 + x_3 & \geq & 90 & (3) \\
x_1 + 2x_3 & \leq & 120 & (4) \\
x_1,\ x_2,\ x_3 & \geq & 0 &
\end{array}
\end{aligned}
$$

The optimal solution of the linear program is presented in the following Simplex Tableau. The variables e_1, e_2, e_3, and e_4 represent the slack and the excess variables used in each constraint and the variable a_3 is the artificial variables used in constraint 3.

Basic Variables	Z	x_1	x_2	x_3	e_1	e_2	e_3	a_3	e_4	Value
Z	1	0	1	0	1	0	0	M	1	220
e_3	0	0	-1	0	1	0	1	-1	0	10
e_2	0	0	-6	0	7	1	0	0	-3	420
x_1	0	1	-2	0	2	0	0	0	-1	80
x_3	0	0	1	1	-1	0	0	0	1	20

(a) (5 points)

Write the dual formulation of this problem.

$$\max\ Z = 2x_1 - 2x_2 + 3x_3$$
subject to:
$$\begin{aligned} x_1 - x_2 + x_3 &\leq 100 && (1)\\ -4_1 + x_2 - x_3 &\leq 80 && (2)\\ x_1 + x_3 &\geq 90 && (3)\\ x_1 + 2x_3 &\leq 120 && (4)\\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

Basic Variables	Z	x_1	x_2	x_3	e_1	e_2	e_3	a_3	e_4	Value
Z	1	0	1	0	1	0	0	M	1	220
e_3	0	0	-1	0	1	0	1	-1	0	10
e_2	0	0	-6	0	7	1	0	0	-3	420
x_1	0	1	-2	0	2	0	0	0	-1	80
x_3	0	0	1	1	-1	0	0	0	1	20

(b) (5 points)

Based on the Simplex Tableau of the optimal primal solution, give the optimal solution of the dual; i.e. the value of the variables as well as the value of the objective function. Justify your approach.

(c) (5 points)

For which values of the coefficient of x_1 in the objective function (i.e. $c_1 = 2$) does the current solution remain optimal? Justify your answer.

$$\max\ Z = 2x_1 - 2x_2 + 3x_3$$

subject to:

$$\begin{aligned} x_1 - x_2 + x_3 &\leq 100 \quad (1) \\ -4_1 + x_2 - x_3 &\leq 80 \quad (2) \\ x_1 + x_3 &\geq 90 \quad (3) \\ x_1 + 2x_3 &\leq 120 \quad (4) \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

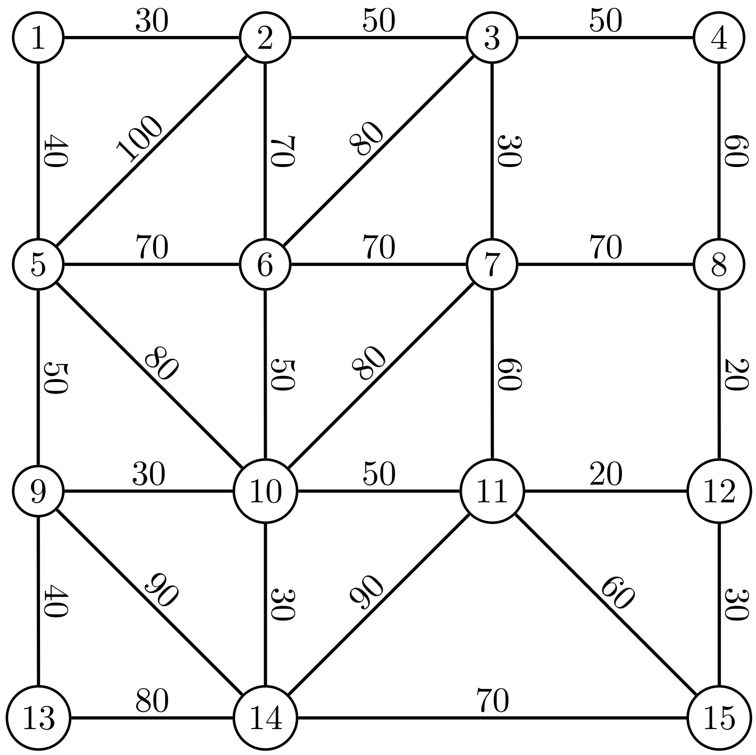
Basic Variables	Z	x_1	x_2	x_3	e_1	e_2	e_3	a_3	e_4	Value
Z	1	0	1	0	1	0	0	M	1	220
e_3	0	0	-1	0	1	0	1	-1	0	10
e_2	0	0	-6	0	7	1	0	0	-3	420
x_1	0	1	-2	0	2	0	0	0	-1	80
x_3	0	0	1	1	-1	0	0	0	1	20

(d) (5 points)

If the right-hand side of the fourth constraint ($b_4 = 120$) increased by 40 units (i.e. $b_4 = 160$), what will be the impact on the value of variables as well as the value of Z?

Question no 6 (20 points)

The nodes of the graph in the following figure represent the 15 future attractions of a theme park. The edges between the nodes represent potential road segments and the number above each edge represents the distance in meters.

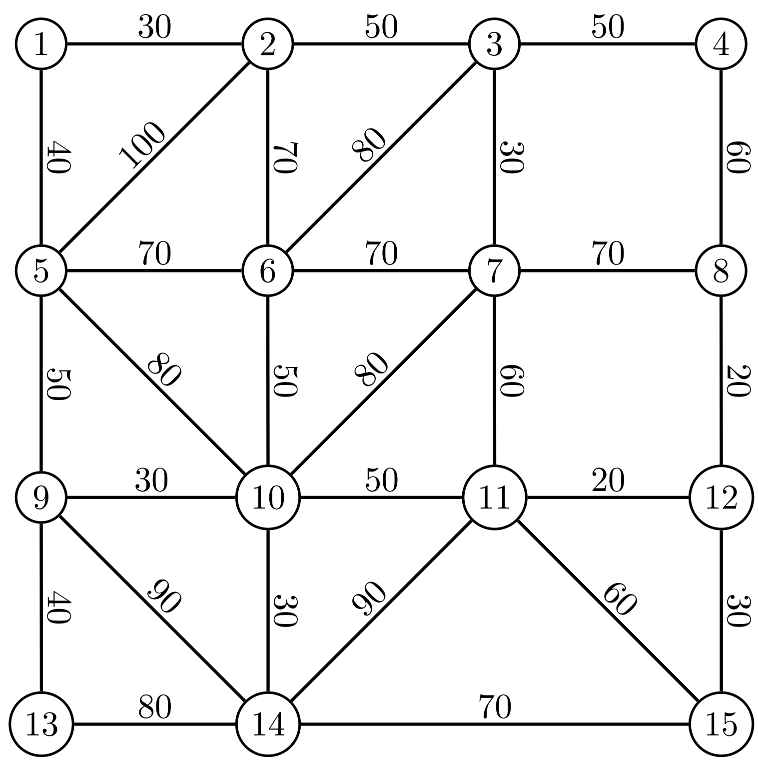


(a) (8 points)

If all road segments were built, what would be the minimum distance from attraction 1 to other attractions?

(b) (8 points)

The park owners are looking to identify which segments they should build iamong those illustrated in the figure in order to connect all the attractions at minimal cost if the construction costs are estimated at \$1000/m. Illustrate the solution on the graph and indicate what will be the cost of this construction.



(c) (4 points)

If the road segment $(9,10)$ can not be built, is the solution found in (b) still valid? If no, indicate which edge will replace $(9,10)$ and what will be the additional construction costs.