ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2015 SESSION

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| Open-book examination<br>Calculators : only authorized models<br>Duration : 3 hours |

14-AL-A3
APPLIED FLUID MECHANICS

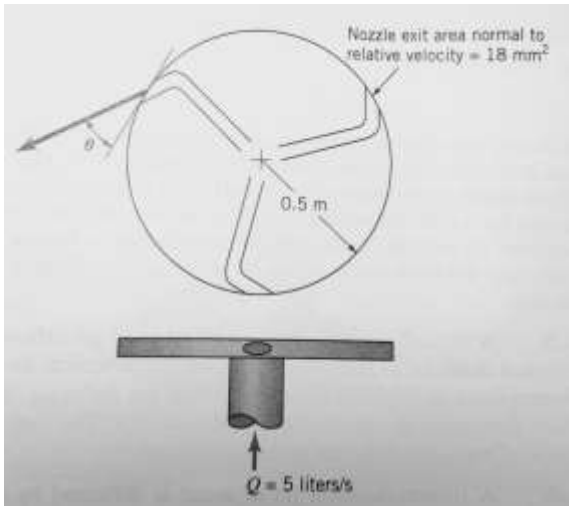
## 1) Rotary water sprinkler: (20 points)

As shown in the following figure, water $\left(\rho = 1000 kg/m^3\right)$ enters the rotor along the axis of rotation at a flow rate of five liters per second (5 l/s). The cross-sectional area of each of the three nozzle exits, normal to the relative velocity, is 18 mm$^2$ and the radius, *r*, from the axis of rotation to the centerline of each nozzle is 500 mm.

How fast will the rotor spin steadily [$\omega_{rotor}$ (rpm)] if the resisting torque is reduced to zero for the two following cases?

**a)** $\theta = 0^o$. **(13 points)**

**b)** $\theta = 60^o$. **(7 points)**



## 2) Flow of a film on the outside of a circular tube: (30 points)

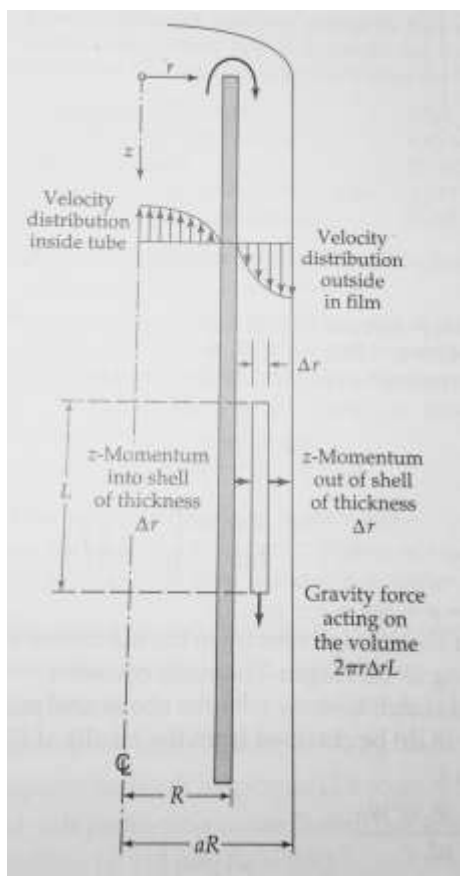
In a gas absorption experiment, a viscous fluid (density: $\rho$ , viscosity: $\mu$ ) flows upward through a small circular tube of radius $R$ and then downward in a laminar flow on the outside of the tube in the form of an annular film of thickness $R(a\text{-}1)$, as shown in the following figure. End effects are considered negligible.

**a)** Set up a momentum balance over a shell thickness Δr in the film, as shown in the figure. **(8 points)**

**b)** Apply the two boundary conditions (at the outside of the tube, i.e. at the interface tube/film, and at the outside surface of the falling film, i.e. at the interface film/air) then show that the velocity distribution inside the falling film is given by the following equation. **(22 points)**

$$v_z(r) = \frac{\rho g R^2}{4\mu}\left[1-\left(\frac{r}{R}\right)^2 + 2a^2 \ln\left(\frac{r}{R}\right)\right]$$

where $g$ is the gravity acceleration and $r$ is the radial position inside the annular falling film.

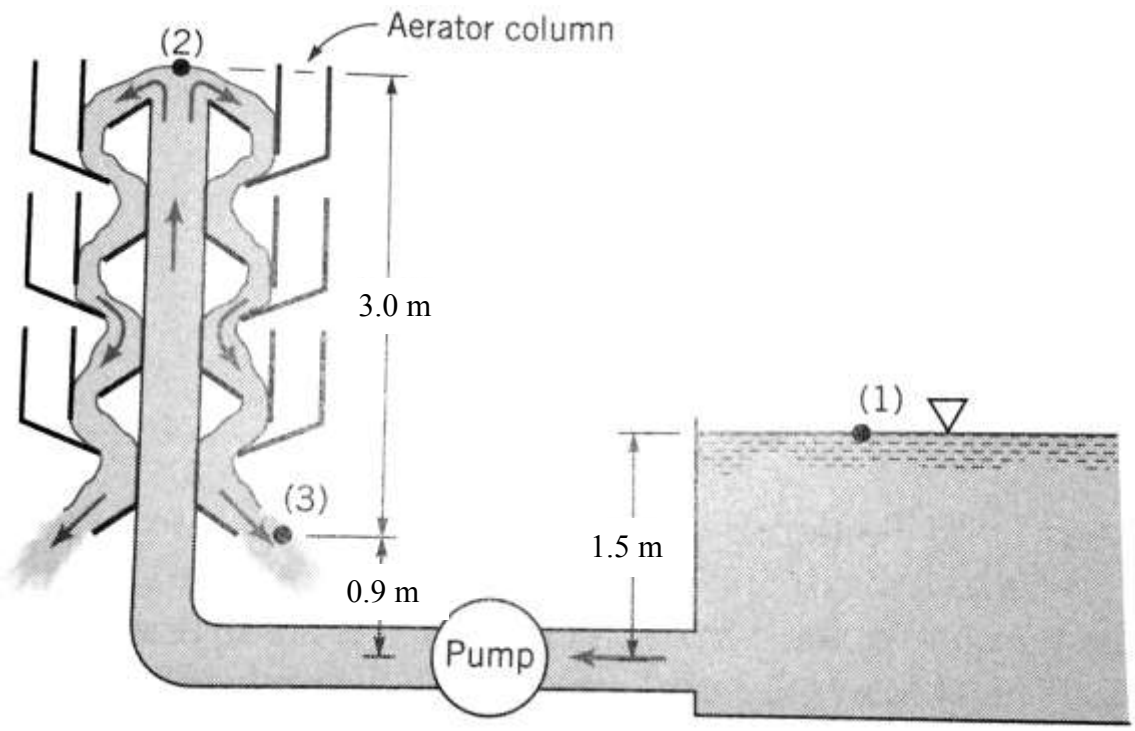


## 3) Water pumping (application of Bernouilli equation): (30 points)

Water $\left(\rho = 1000 kg/m^3\right)$ is pumped at room temperature at a volumetric flow rate of $85 \times 10^{-3}$ $m^3/s$ from a tank, point (1), to the top of a water plant aerator, point (2), as shown in the following figure.

**a)** Determine the power $\dot{W}(Watt)$ that the pump adds to the water if the head loss $h_L$ from (1) to (2) where $v_2 = 0$ is 1.22 m. **(15 points)**

**b)** Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is $v_3 = 0.6$ m/s. **(15 points)**



## 4) Hydraulic jump: (20 points)

Water discharging into a 10 m wide rectangular horizontal channel from a sluice gate is observed to have undergone a hydraulic jump, as shown in the following figure. The flow depth and velocity before the jump are 0.8m and 7m/s, respectively. The density of water is $\rho = 1000 kg/m^3$.

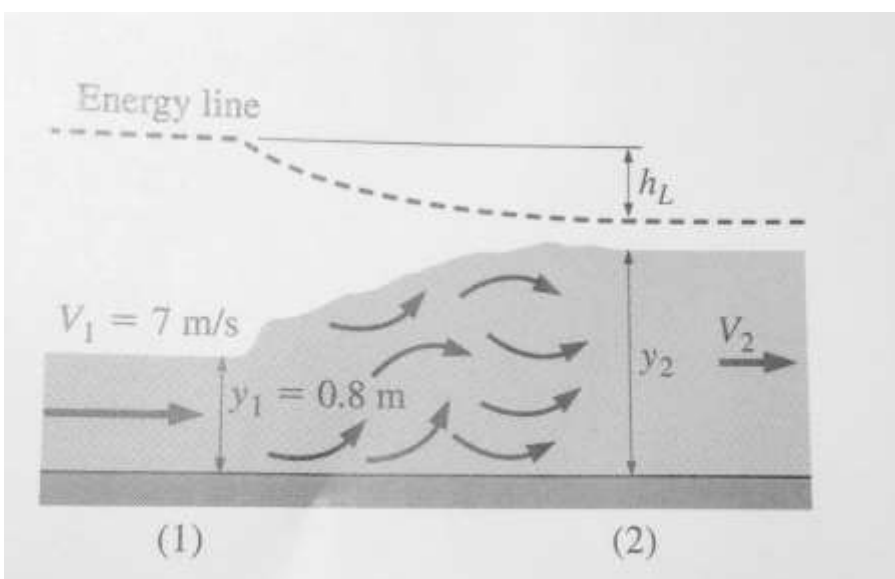
Assume that the flow is steady and the channel is sufficiently wide so that the end effects are negligible.

**a)** Under what condition (in terms of the Froude number before the jump, $Fr_1$) the hydraulic jump takes place? **(4 points)**

**b)** Determine the flow depth, $y_2$, and the Froude number, $Fr_2$, after the jump. **(8 points)**

**c)** Determine the head loss, $h_L$ (m), and energy dissipation ratio, $h_L/E_{s1}$, where $E_{s1}$ is the specific energy of water before the jump. **(8 points)**

Nb : For a flow through a rectangular channel, the Froude number is expressed in terms of the average flow velocity $\langle V \rangle$ and the depth $H$ of the channel by the following relation: $Fr = \frac{\langle V \rangle}{\sqrt{g \cdot H}}$, where g= 9.81 m/s$^2$.



# Appendixes

## 1. Generalized Newton's law of viscosity:

*Cylindrical coordinates $(r, \theta, z)$:*

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-8)}^a$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-9)}^a$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-10)}^a$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad \text{(B.1-11)}$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right] \qquad \text{(B.1-12)}$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right] \qquad \text{(B.1-13)}$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \qquad \text{(B.1-14)}$$

## 2. Continuity equation:

*Cartesian coordinates $(x, y, z)$:*

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-1)}$$

*Cylindrical coordinates $(r, \theta, z)$:*

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-2)}$$

*Spherical coordinates $(r, \theta, \phi)$:*

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \qquad \text{(B.4-3)}$$

## 3. Equation of motion for Newtonian fluids (Navier-Stokes equation):

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

*Cylindrical coordinates* $(r, \theta, z)$:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r \quad \text{(B.6-4)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta \quad \text{(B.6-5)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-6)}$$

## 4. Moody diagram:

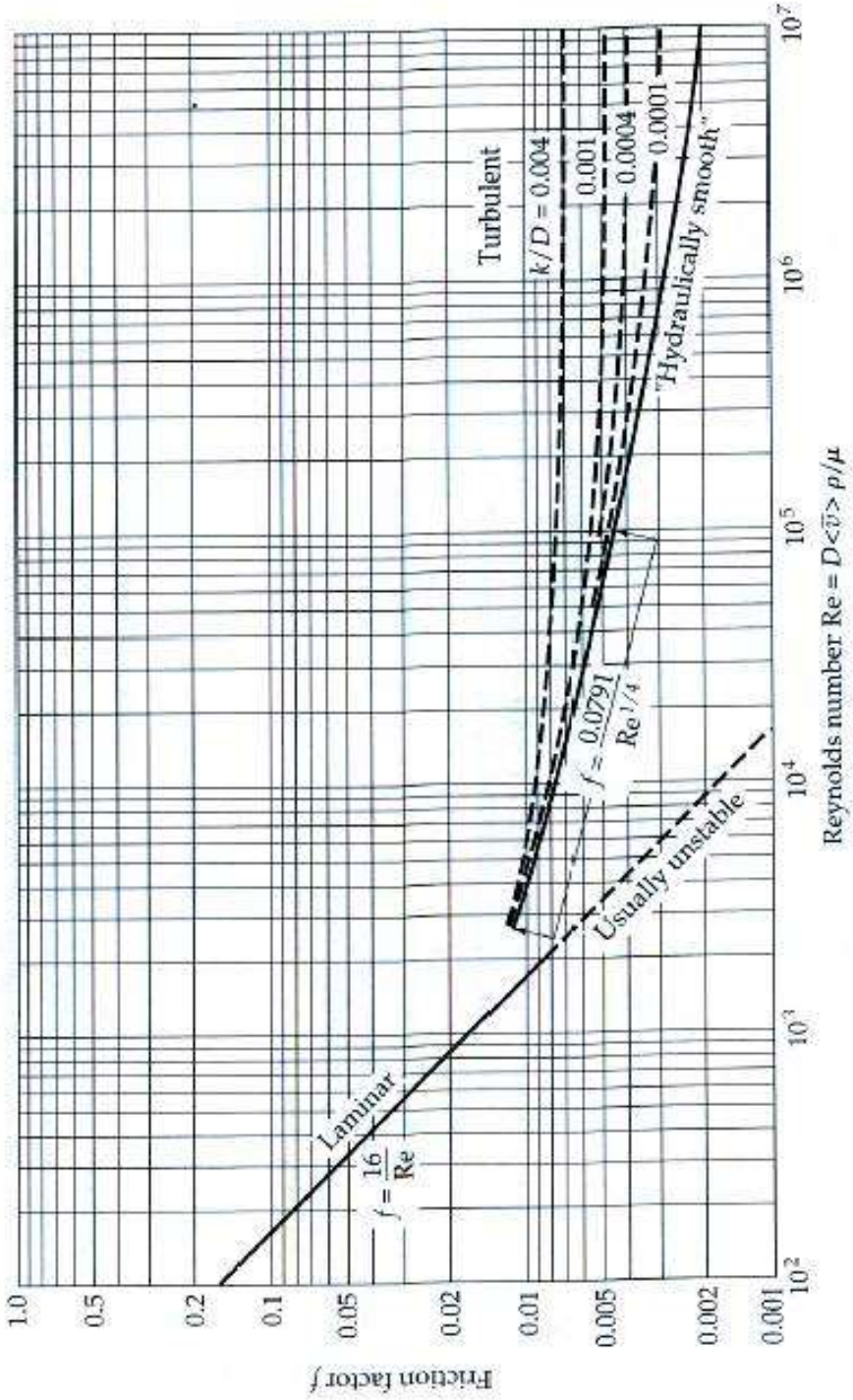



**Fig. 6.2-2.** Friction factor for tube flow (see definition of $f$ in Eqs. 6.1-2 and 6.1-3. [Curves of L. F. Moody, *Trans. ASME*, **66**, 671–684 (1944) as presented in W. L. McCabe and J. C. Smith, *Unit Operations of Chemical Engineering*, McGraw-Hill, New York (1954).]