ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2018 SESSION

Open-book examination Calculators: Only authorized models Duration : 3 hours

14-AE-A2
APPLIED FLUID MECHANICS

1) Newtonian and non-Newtonian fluids: (20 points)

a) For each of the following fluids, indicate if it is Newtonian, Yield-stress (Bigham), shear thickening or shear thinning: i) Cooking oil; ii) Jello; iii) Mayonnaise; iv) Milk 2%; and v) Latex paint. **(5 points)**

b) Toluene is inserted at room temperature between two large parallel plates (plate area = 5m^2) separated by a distance of 10mm. The two plates are set in motion in opposite directions: the upper one is moved in the positive direction at a constant speed of 0.3 m/s by applying a force of 0.83 N and the lower one in the negative direction at a constant speed of 0.1 m/s. Deduce the dynamic viscosity of toluene at room temperature. **(8 points)**

c) What is the state of a Newtonian fluid (liquid, gas or solid!) at a reduced temperature $T_r = T/T_c > 1.0$? (T_c is the critical temperature of the fluid). **(4 points)**

d) What will be its state if this temperature is maintained constant and the pressure is increased to a value higher than its critical pressure, p_c? **(3 points)**

2) Sheathing of cables: (30 points)

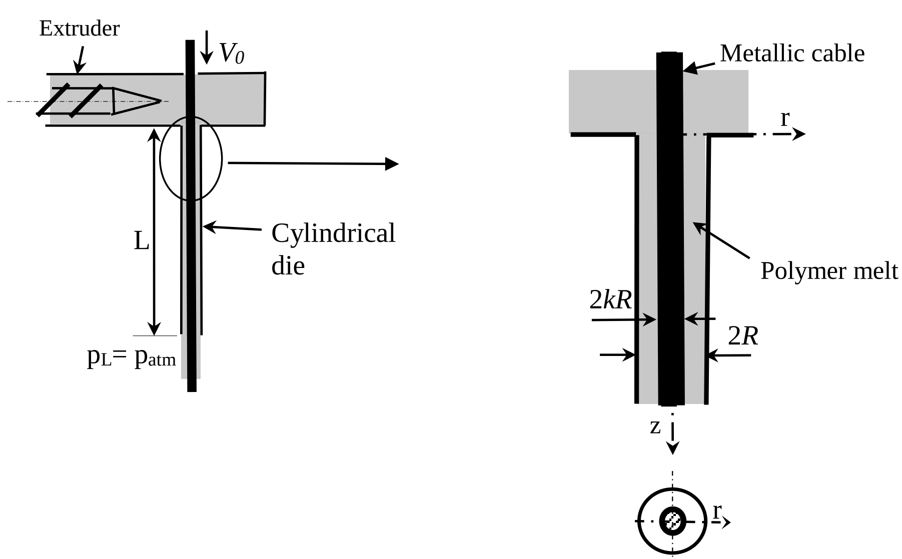
To be protected against corrosion, a steel cable of radius kR (k <1.0) must be covered with a thin layer of thermoplastic pumped in melt state by an extruder into a cylindrical cable die (length: L; radius: R). The cable is pulled at a constant speed V_0 through the die, as illustrated by the sketch below. The pressure imposed by the extruder at the entrance of the die is $p_0 > p_{atm}$ and that at the exit of the die is $P_L = p_{atm}$ (p_{atm} is the atmospheric pressure).

The melt polymer is considered as Newtonian with a constant density ρ and constant dynamic viscosity μ.

a) Get the two required boundary conditions (at the interfaces cable/polymer and polymer/die) and simplify the equation of continuity and the three components of the equation of motion (Navier-Stokes equation) presented in the appendix. **(10 points)**

b) Find the velocity distribution of the polymer melt in the annular region between the cable and the inner surface of the cylindrical die. **(12 points)**

c) Obtain the necessary pulling force on the cable in order to keep a constant cable velocity V_0. **(8 points)**



3) Water transfer from a well to a tank: (30 points)

The irrigation of an area requires water supply from a well at a flow rate of 40 l/s by using a centrifugal pump, as shown in the following figure. The suction pipe (upstream of the pump) has a diameter of 200mm, a total length of 8m and includes i) a foot valve strainer (Loss coefficient, K= 3.5); (ii) 135^o elbow (K= 0.15); and (iii) a converging conical pump inlet (K = 0.1).

The discharge pipe (downstream of the pump) has a diameter of 175mm and total length of 160m. It includes: i) a diverging conical pump outlet (K= 0.25); (ii) an open valve (K= 0.2); (iii) three 90^o elbows (K= 0.2); (iv) a check valve (K= 1.5); and (v) an outlet to the upper tank (K= 0.5).

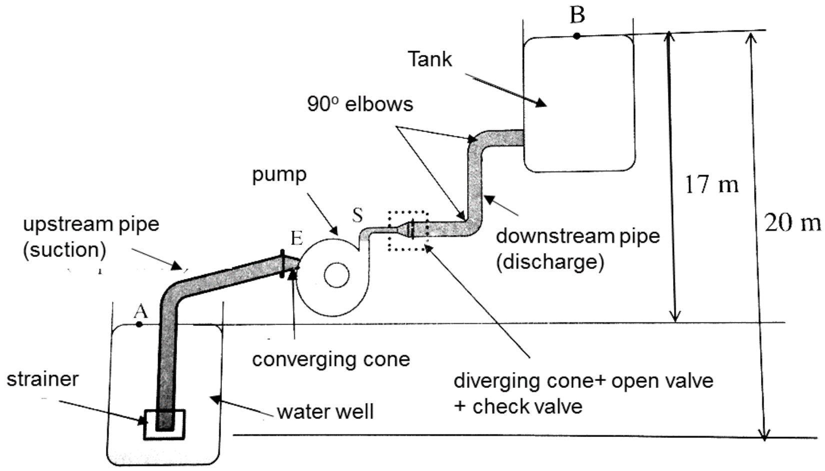
As shown in the figure, the total geometric head (height between levels A and B) is 17 m and the suction geometric height is equal to 3m.

All pipes are made from galvanized steel and have an absolute roughness ε = 0.15 mm. Under ambient conditions, water has a density ρ = l000kg/m^3 and a dynamic viscosity μ = $l0^{-3}$ Pa.s.

a) Use Bernoulli equation between the two levels A and B to calculate the net height H_n (m) that should provide the pump to maintain the constant flow rate of 40 1/s. **(13 points)**

b) Use Bernoulli equation between levels A and E then between points S and B to calculate the effective pressures in terms of height at point E (pump inlet) and point S (pump outlet). **(12 points)**

c) If the pump is driven by an electric motor providing a power of 12 kW, what is its efficiency? **(5 points)**

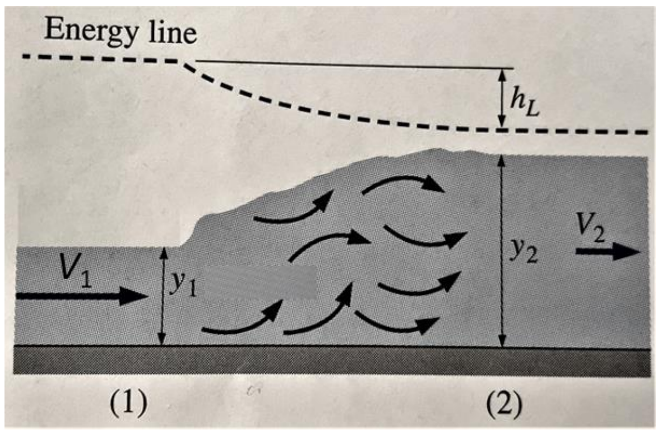


4) Water discharging into a rectangular horizontal channel from a sluice gate: (20 points)

Water discharging into a rectangular horizontal channel from a sluice gate is observed to have undergone a hydraulic jump, as shown in the following figure. By considering a unit width of the channel, use the continuity, the momentum and energy equations to:

a) Determine the velocity V_2 and the depth y_2 after hydraulic jump as a function of the rectangular channel upstream conditions. **(12 points)**

b) Deduce the head loss, h_L, associated to the hydraulic jump. **(8 points)**



APPENDIX

(if needed)

1. Generalized Newtonian Law of Viscosity:

$$[\boldsymbol{\tau} = -\mu(\nabla \mathbf{v} + (\nabla \mathbf{v})^{\dagger}) + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})\boldsymbol{\delta}]$$

Cylindrical coordinates (r, θ, z):

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-8)}^a$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-9)}^a$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-10)}^a$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad \text{(B.1-11)}$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right] \qquad \text{(B.1-12)}$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right] \qquad \text{(B.1-13)}$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \qquad \text{(B.1-14)}$$

2. Continuity equation:

Cartesian coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-1)}$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-2)}$$

Spherical coordinates (r, θ, ϕ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \qquad \text{(B.4-3)}$$

3. Navier-Stokes Equation:

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r \quad \text{(B.6-4)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta \quad \text{(B.6-5)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-6)}$$

4. Moody diagram:

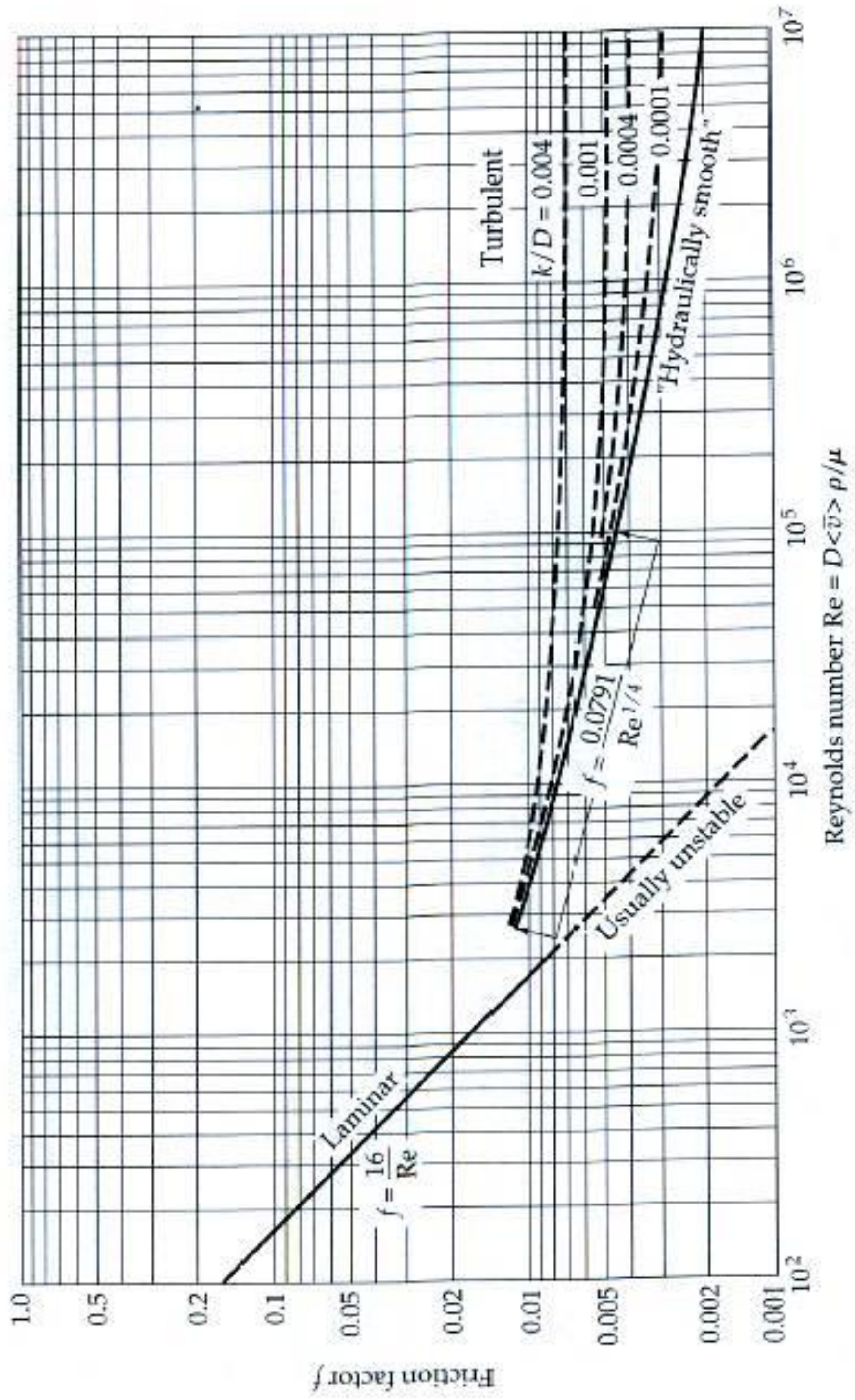


Fig. 6.2-2. Friction factor for tube flow (see definition of f in Eqs. 6.1-2 and 6.1-3. [Curves of L. F. Moody, *Trans. ASME*, **66**, 671–684 (1944) as presented in W. L. McCabe and J. C. Smith, *Unit Operations of Chemical Engineering*, McGraw-Hill, New York (1954).]