ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2019 SESSION

Open-book examination Calculators: Only authorized models Duration : 3 hours

14-AE-A2
APPLIED FLUID MECHANICS

1) Newtonian and non-Newtonian fluids: (20 points)

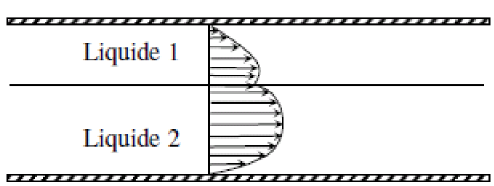
a) Describe each of the following fluids: Shear thickening, Thixotropic, Rheopectic, Bingham, Magnetorheological. Give an example for each one. **(5 points)**

b) Explain why the dynamic viscosity of low-density gases increases with increasing temperature, however that of liquids decreases. **(4 points)**

c) What is the state of a Newtonian fluid (liquid, gas or solid!) at a reduced temperature $T_r = T/T_c > 1.0$? (T_c is the critical temperature of the fluid). Under this condition, what is the effect of increasing the pressure? **(4 points)**

d) What are the two mechanisms of momentum transport during the flow of Newtonian fluids? Give a brief description of each one of these two mechanisms. **(4 points)**

e) Two immiscible Newtonian liquids, 1 and 2, are flowing in laminar flow between two parallel plates (sketch below). Is it possible that the velocity profiles would be of the following form? Explain. **(3 points)**



2) <u>Slow rotation of a cylinder in a Newtonian fluid:</u> **(30 points)**

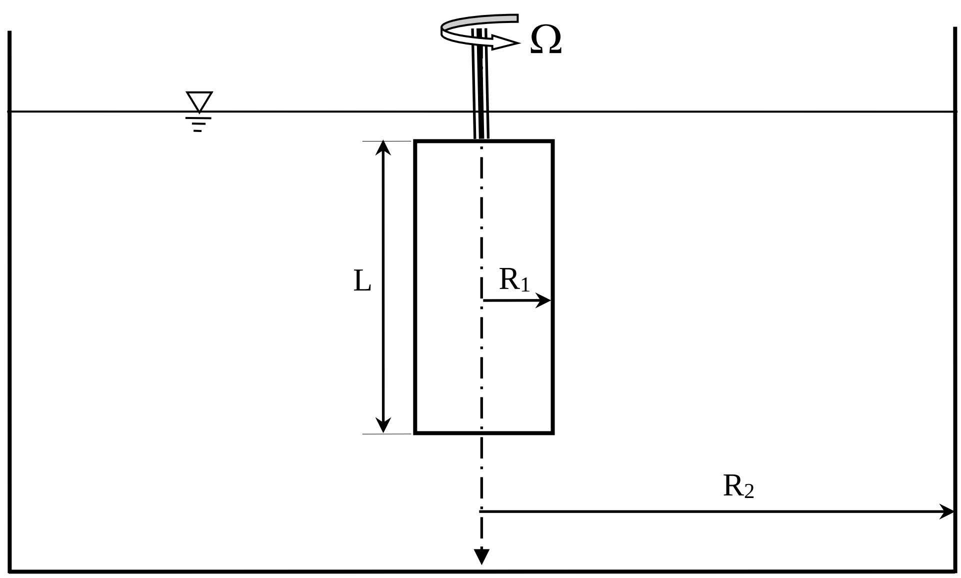
A cylindrical mixer (radius: R_1; length: L) is rotating at a slow rotational speed Ω (rad/s) inside an incompressible Newtonian fluid of dynamic viscosity μ contained in a large tank of radius $R_2 \gg R_1$ (i.e., $\frac{R_1}{R_2} \approx 0$) (see sketch below).

a) What are the two boundary conditions needed to calculate the fluid velocity and the shearing stress profiles during the steady-state rotation of the inner cylinder? **(4 points)**

b) Under steady-state conditions, simplify the continuity equation and the three components of the Navier-Stokes equation given in the appendix. Consider that, for a slow rotational speed, the centrifugal forces are neglected. **(6 points)**

c) Demonstrate that the tangential velocity profile is given by the following relation $v_\theta = \Omega \frac{R_1^2}{r}$ where r is the radial position from the rotational axis. **(12 points)**

d) Get the expression of the torque required to rotate the cylinder at a constant speed Ω. **(8 points)**

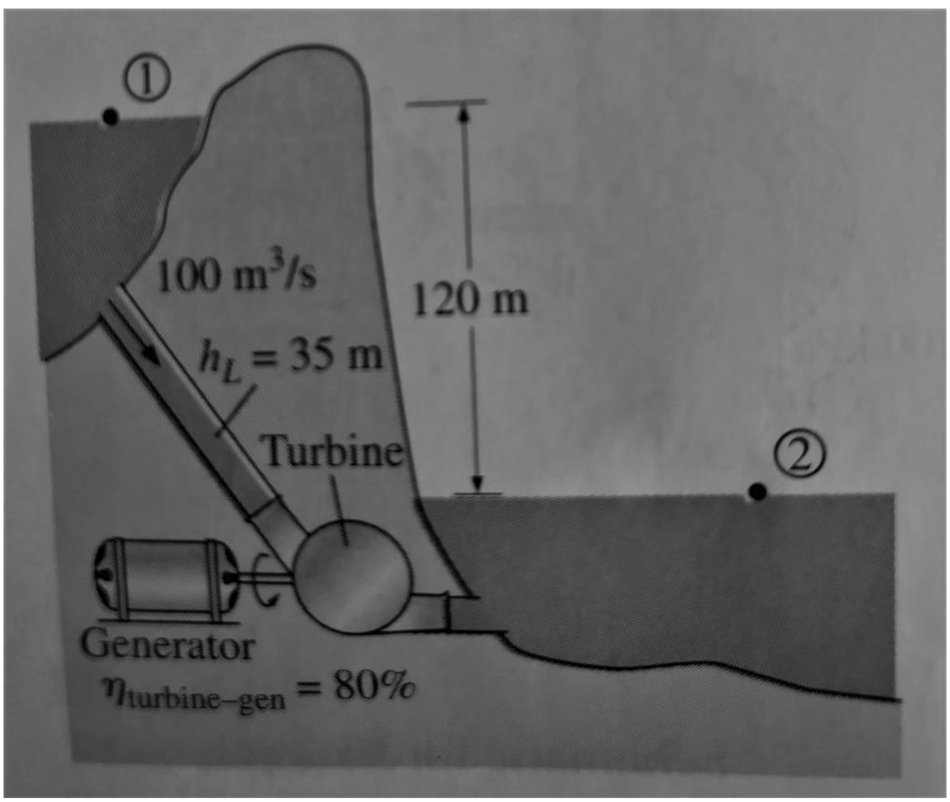


3) Hydraulic power generation from a Dam: (20 points)

In a hydraulic power plant, water (ρ = l000kg/m^3) flows at a rate of 100m^3/s from an upper reservoir (elevation of 120 m) to a turbine, where electric power is generated. The total irreversible head loss h_L in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m.

a) Use the macroscopic energy equation to calculate the extracted turbine head $h_{turbine,e}$. **(10 points)**

b) Deduce the perfect turbine-generator electric power output $\dot{W}_{turbine}$ and if its overall efficiency is 80%, estimate its real electric power output. **(10 points)**



4) Hydraulic jump: (30 points)

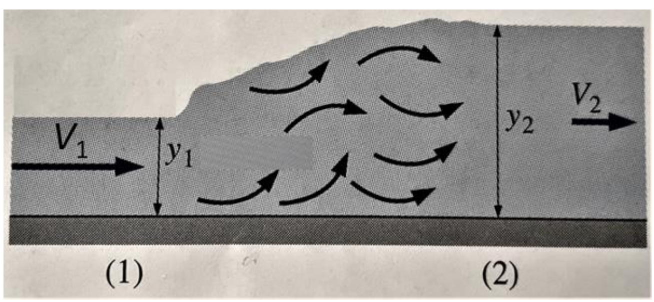
Consider the flow of water (ρ = 1000kg/m^3) in a 10-m-wide channel at a rate of 70 m^3/s and a flow depth y_1 of 0.50 m. The water undergoes a hydraulic jump, and the flow depth y_2 after the jump is measured to be 4 m.

a) Determine the velocity V_2 after the hydraulic jump. **(7 points)**

b) Determine the head loss, h_L , from the energy equation. **(7 points)**

c) Determine the specific energy of water before the jump and the energy dissipation ratio. **(8 points)**

d) Determine the mechanical power wasted during this jump. **(8 points)**



APPENDIX

1. Generalized Newtonian Law of Viscosity:

$$[\boldsymbol{\tau} = -\mu(\nabla \mathbf{v} + (\nabla \mathbf{v})^{\dagger}) + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})\boldsymbol{\delta}]$$

Cylindrical coordinates (r, θ, z):

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-8)}^a$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-9)}^a$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-10)}^a$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad \text{(B.1-11)}$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right] \qquad \text{(B.1-12)}$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right] \qquad \text{(B.1-13)}$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \qquad \text{(B.1-14)}$$

2. Continuity Equation:

Cartesian coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-1)}$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-2)}$$

Spherical coordinates (r, θ, ϕ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \qquad \text{(B.4-3)}$$

3. Navier-Stokes Equation:

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g}]$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r \tag{B.6-4}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta \tag{B.6-5}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \tag{B.6-6}$$