ORDRE DES INGÉNIEURS DU QUÉBEC

NOVEMBRE 2022 SESSION

Open book examination
Calculator : authorized model only
Duration : 3 hours

20-MB-A1 MATHEMATICS

Question 1. (10+10+10=30 points)

Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

a) Determine the eigenvalues of A and corresponding eigenvectors.

b) What are the eigenvalues of A^2 ?

c) Compute the inverse matrix A^{-1}.

Question 2. (10+10=20 points)

a) Find the general form of the solution of the following differential equation, knowing that e^x is a solution.

$$xy''(x) - (x+1)y'(x) + y(x) = 0.$$

b) Find the general form of the solution of the following differential equation.

$$x^2 y''(x) - 2y(x) = x^2.$$

Suggestion : first, find solutions of the homogeneous equation in the form $y(x) = x^a$ for some $a \in \mathbb{R}$.

Question 3. (10+10=20 points)

a) Do the following series converge or not (justify your responses).

$$\sum_{n=1}^{+\infty} \frac{1}{n+2^n}.$$

$$\sum_{n=2}^{+\infty} \frac{2^n}{1 \cdot 3 \cdots (2n-1)}.$$

b) Find the radius of convergence of the following series (justify).

$$\sum_{n=1}^{+\infty} \frac{x^n}{(2n)!}.$$

$$\sum_{n=1}^{+\infty} \frac{(2x+1)n}{n^2}.$$

Question 4. (10+10+10=30 points)

a) What is the curvature of a circle of radius R? (justify).

b) Find the length of the curve $r(t) = (t, \cosh t)$ from $t = 0$ to $t = 1$.

c) Using a parametric representation of a sphere of radius R, show that the surface area is $4\pi R^2$.